NICOLET SCIENTIFIC CORPORATION

MODEL 660

DUAL-CHANNEL FFT ANALYZER

NICOLET SCIENTIFIC CORPORATION
(formerly Federal Scientific Corporation)
245 LIVINGSTON STREET, NORTHVALE, NEW JERSEY 07647

EXTENDER BOARDS

SMALL

9280025 #13294

LARGE

9280143 #97700

FRONT PANEL POWER SWITCH

P/N 22-00017 SPO 92

NO. ON SWITCH ITSELF 01-700265

NICOLET SCIENTIFIC CORPORATION

Rep. DEAN DAWSON (714) 261 549-7611
HOME (714) 552-0664 Serial Number 70 66 041

Instruction Manual

for

MODEL 660

DUAL-CHANNEL FFT ANALYZER

NICOLET SCIENTIFIC CORPORATION

NICOLET SCIENTIFIC CORPORATION / 245 LIVINGSTON ST., NORTHVALE, N. J. 07647 (201) 767-7100
(formerly Federal Scientific Corporation)

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manufacturers of UBIQUITOUS® instruments

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Dear Customer:

I would like to thank you personally for having selected Nicolet Scientific Corporation equipment for your application. Most instruments that we produce are burned-in and subjected to vibration and temperature tests before shipment, to maximize the equipment's reliability.

If, however, your equipment should not operate properly upon receipt, check your interconnections and control settings against the information supplied in this instruction manual. Should you find that the equipment fails to operate as specified, call us. We will either provide service information via phone conversation or we will authorize your return of the equipment for factory servicing, if required.

May I ask your cooperation in completing the enclosed, prepaid postcards? The information which we request will permit us to serve you better in the future. We would like to know to whom to address any Instruction Manual revisions, field modifications, suggestions, or new application literature.

Thank you again for purchasing our equipment. We look forward to serving you again in the future.

Sincerely,

Henry J. Bickel
President

F O R W A R D

This manual provides operating instructions for the Model 660 Dual-Channel FFT Processor. These instructions introduce the Model 660 controls on a function-group/by/function-group basis. The order of control introduction follows the flow of selections in normal operation. It is recommended that these instructions be read in their entirety before attempting to operate the equipment.

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SPECIAL INSTRUCTIONS

SPECIAL INSTRUCTIONS

WARNING

CONCERNING SAFE OPERATION OF ELECTRICAL EQUIPMENT:

1. This equipment is supplied with a detachable, rubber-covered electrical ac power line cord. One termination of the cord is a standard three-contact female connector for mating the AC INPUT receptacle on the instrument's rear panel. The other termination is a 3-prong male connector for mating a power source outlet, with one of the power source terminals at ground potential. THIS EQUIPMENT SHOULD NEVER BE OPERATED UNLESS IT IS PROPERLY GROUNDED.

2. If you use a 2-prong to 3-prong adapter for mating to your power source outlet, make sure that the pigtail lead (the grounding strap which is provided on the 2-prong adapter) is securely grounded. FAILURE TO FOLLOW THIS WARNING MAY RESULT IN ELECTRICAL SHOCK TO OPERATING AND MAINTENANCE PERSONNEL.

3. Nominal 115-volt or 220-volt ac potentials are present on the terminals of the front panel POWER switch and on the rear panel ac POWER INPUT connector and line protective fuse. Nominal 115-volt or 220-volt ac potentials also exist on the terminal strip of the self-contained power supply module and at various electrical terminals within the power supply module. ALWAYS EXERCISE EXTREME CARE WHEN WORKING IN OR NEAR THESE AREAS TO AVOID SEVERE ELECTRICAL SHOCK!

4. Lethal dc voltages are present within all Nicolet Scientific Corp. manufactured equipment utilizing switching type power supplies and CRT displays. This equipment should only be serviced by TRAINED PERSONNEL using extreme care to avoid severe electrical shock.

SECTION I

OVERALL SPECIFICATIONS

FUNCTIONS CALCULATED AND DISPLAYED (19)

1. Input time history (dual)
2. Ensemble averaged input time history (SIG ENH) (dual)
3. Instant Spectrum of input time or averaged input time
   (a) 400-line (dual) – INST SPEC, 1K CH A-B
   (b) 800-line (single) – INST SPEC, 2K CH A
4. Power spectral density - PSD (dual) – POWER SPEC, PER Hz
5. Averaged power spectrum (dual) – POWER SPEC
6. RMS spectrum (square root of averaged power spectrum (dual) – RMS SPEC
7. Cross-spectrum averaged (complex-amplitude/phase or real/imaginary) – CROSS SPEC
8. Transfer function (complex-amplitude/phase or real/imaginary) – TF
9. Equalized transfer function
10. Transmissibility (ratio of RMS spectrum B to RMS spectrum A) – TM
11. Equalized transmissibility
12. Coherence (COH)
13. Coherent Output Power (COP)
14. Impulse response – IMPUL RESP
15. Equalized impulse response
16. Probability Density Function (PD) – PROB DEN
17. Cumulative Distribution Function (CD) – CUM DIST
18. Auto-correlation – AUTO CORR
19. Cross-correlation – CROSS CORR

STORAGE OF FUNCTIONS

STORE - Any dual display (dual-channel or cross-function, such as TF, real and imaginary or magnitude and phase) can be stored together with annotation; a single time function or one 800-pt. spectrum can be stored with annotation.

RECALL - Place stored data on display screen (LED on); revert back to current data display upon second actuation of pushbutton (LED off).

COMPare - One set of stored data and current data displayed simultaneously (CH A and CH A', TF magnitude stored and TF magnitude current, etc.).

EQL TF/TM - Equalizes transfer function if TF is stored (real and imaginary or magnitude and phase) by dividing magnitude of current by stored and subtracting phase of stored from current; also compensates impulse response and TM.

INPUT CHARACTERISTICS (Each Input Channel)

Maximum Amplitude (Volts 0 - Peak): 0.1, 0.2, 0.5, 1, 2, 5, 10, 20

Coupling: AC or DC, normal or inverted, ZERO grounds input; AC -3 dB at 0.5 Hz

Impedance: 100K ohms

Test Signal: Single peak approximately full scale appears in each channel at a frequency which is 64 percent of analysis range (normal input is disconnected when test signal is on).

ANALYSIS CHARACTERISTICS

Frequency (Time Ranges): 16 ranges from 0 - 1 Hz to 0 - 100 kHz in 1-2-5 steps (400 sec to 4 ms for 2-ch, 800 sec to 8 ms for 1-ch)

Resolution: Samples in time functions - 1024 for each of 2-ch, 2048 for 1-ch. Resolution elements in frequency functions - 400 for each of 2-ch, 800 for 1-ch.

Input Sampling: 12-bit A/D conversion at 2.56 times maximum frequency in the range selected or 256,000 per second maximum.

Input Filters: Range switch automatically selects a pair of matched, low-pass, anti-aliasing filters, which provide at least 70 dB attenuation of those input frequencies which can cause spurious responses due to aliasing (10 Hz filter used on 1, 2 and 5 Hz ranges), frequency response ±0.5 dB (except ±1.0 dB on 100 kHz range)

Transient Capture Control: Digital data in input buffer memories may be held either manually by placing the HOLD/REL button in HOLD (light on), or automatically by triggering on the transient waveform in either A or B input channels, at a preset trigger level (±1/2, ±1/4, ±1/8, or ±1/16 FS) after ARM is pushed (light on); delay or anticipation of the trigger may be set using the keyboard (T→A) in full or fractional time windows as determined by the frequency range setting.

Input Amplitude Monitors: Lights indicate peak signal levels - OVERLOAD, -6 dB (greater than 1/2 full scale), -12 dB (between 1/2 and 1/4 FS), -18 dB (between 1/4 and 1/8 FS), -24 dB (between 1/8 and 1/16 FS); electronic graticule top and bottom in time display indicates amplitude limit.

BUILT-IN ANNOTATED DISPLAY

Vertical Display Gain: In linear, full scale can be set to 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6 or 8 X 10^K; in log from +300 dB to -300 dB in 10 dB steps, display range set from 10 dB to 80 dB in 10 dB steps via keyboard (DSPL dB).

Time Displays: Self-aligning electronic graticule, input maximum amplitude written on CRT in volts 0 - peak.

Frequency Displays: Self-aligning electronic graticule, numbers indicate input maximum amplitude of each channel, full scale frequency, averaging mode and number of spectra averaged, type of function, format, weighting, cursor amplitude in calibration units, cursor frequency, and latest keyboard-entered data.

DISPLAY FORMATS (where applicable to function selected)

Time: Ch A, ch B, ch A and ch B, correlation, impulse response

Frequency: Ch A, ch B, ch A and ch B.

Cross-Frequency: Real, imaginary, real and imaginary, magnitude, phase, magnitude and phase, Nyquist (real vs imaginary).

Probability: PD ch A, PD ch B, PD ch A and ch B, CD ch A, CD ch B, CD ch A and ch B.

CALIBRATION AT CURSOR

Marker: Single line cursor intensified each waveform at its intersection to indicate location of measurement; in HARMONIC, all multiples of cursor frequency are illuminated to indicate frequencies harmonically related to the cursor position.

Amplitude: In engineering units, using keyboard to establish EU for a reference signal or sensitivity as V/EU; dB reference can be set for relative measurements; readings can be in volts, power (V^2), or dB; readings can be normalized per Hz (such as PSD).

Frequency: In Hz (cycles/sec), CPM (cycles/minute), or orders (enter FS order via keyboard); reads absolute or as difference from reference frequency (FREQ REF set with keyboard).

Time: Seconds or millisecs; %FS in "orders" (external sampling).

CONTROL STORAGE

Stores and recalls nine sets of control settings. Each set includes a selection in each of the following control groups:

CURSOR TYPE	DELAY T→A	DISPLAY FORMAT
CURSOR READ	DELAY A→B	DISPLAY MEMORY
DISPLAY GAIN	MODE	COHERENCE BLANK
AVERAGE MODE	FUNCTION	PLOT RATE
AVERAGE N	DISPLAY CH	DISPLAY dB

KEYBOARD SETTINGS

The following parameters can be set and recalled:

1. **AVG N** - Number of functions averaged
2. **DELAY T→A** - Delay (or anticipation) between trigger signal and start of Channel A data window, in time windows or fraction of windows.
3. **DELAY A→B** - Delay introduced between Channels A and B in time windows.
4. **PANEL** - Stores nine sets of panel settings with identification number for future recall.
5. **RMS 1→2** - Calculates total RMS power between cursor location number 1 and number 2.
6. **$\int dt$** - Synthetic time-domain integration or double integration after processing affects all averaged functions involving the integrated input channel (e.g. converts acceleration to velocity to displacement).
7. **d/dt** - Synthetic time-domain differentiation or double differentiation after processing affects all functions involving the differentiated input channel (e.g. converts displacement to velocity to acceleration).
8. **COH BLANK** - Sets level of display blanking for all spectra based on degree of coherence.
9. **PLOT RATE** - Sets (by code) from very slow to very fast in five steps
10. **DSPL dB** - Sets log scale from 10 to 80 dB full range in 10 dB steps.
11. **LIN (REF)** - The amplitude at cursor is equated to a numerical reference.
12. **dB (REF)** - The amplitude at cursor is set to a dB reference.
13. **V/EU** - Sets calibration sensitivity of amplitude measurements.
14. **FREQ REF** - Sets cursor location as reference for frequency readings.
15. **ORDER REF** - Sets cursor location as particular order number.

AVERAGING

Type:

SUM adds N power spectra

DIFF subtracts N power spectra from pervious average

EX follows the changing statistics of a signal exponentially with a time constant of N X time window

PEAK produces the profile of the maximum amplitudes at each frequency element during N spectra

SWEEP adds power spectra with maximum redundancy until averaging is manually stopped

TRANS averages power spectra of N transients automatically, *except* those which overload input; after each capture the transient trigger is automatically armed.

Control:

START (erases memory and starts averaging)

STOP (halts averaging)

CONT (resumes averaging until N is reached, *or* restarts averaging to N without erasing memory)

Number set using keyboard (AVG N)

DIGITAL INTERFACES (standard built-in):

(1) IEEE Standard 488-1975 Instrumentation Interface, full listener/talker

(2) RS232 Type D Serial Interface, 660 acts as a terminal for transmission via a modem

(3) High speed parallel input/output bus with control of bus via IEEE488 or RS232

AVAILABLE OPTIONS

Digital Recording (Model 134): Produces hardcopy of total annotated display.

Frequency Expansion (-2): Built-in dual channel windows 50, 100, 500, 1000, or 2000 Hz wide, which can be located in 10-Hz steps to 100 kHz (designated "-2 (window), e.g. -2 (100) for 100 Hz window.

Mass Storage (Model 160A) Digital cartridge recorder stores functions and annotation displayed with ID numbers for search and playback.

Modal Analysis (MA): Records view of test object and displays dynamic vibrations in each mode.

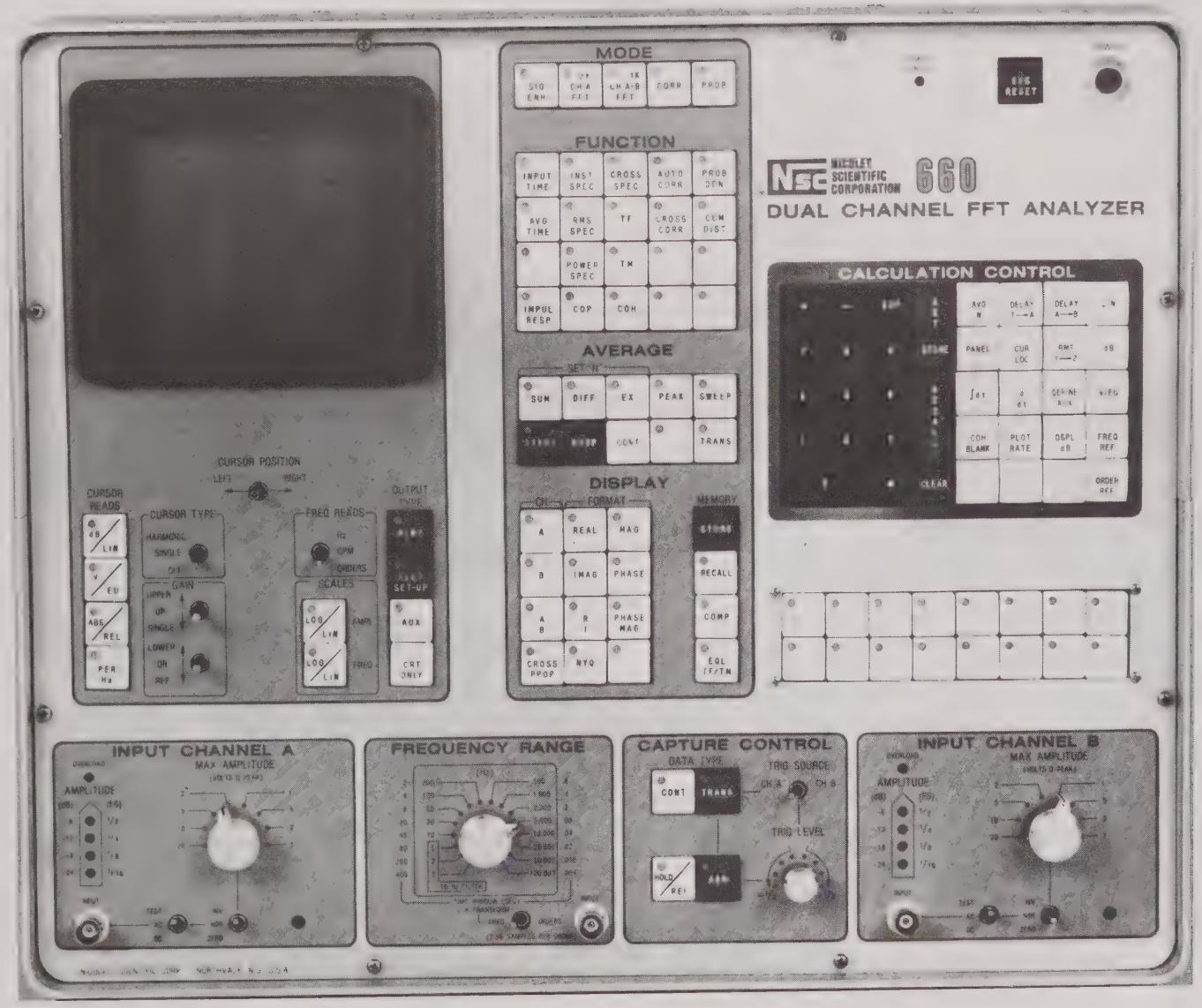

MODE
FUNCTION
AVERAGE
DISPLAY
NSC
HEWLETT SCIENTIFIC CORPORATION
660
DUAL CHANNEL FFT ANALYZER
CALCULATION CONTROL
CURSOR POSITION
CURSOR READS
CURSOR TYPE
FREQ READS
GAIN
SCALES
OUTPUT TYPE
INPUT CHANNEL A
FREQUENCY RANGE
CAPTURE CONTROL
INPUT CHANNEL B

FIGURE 2A

SECTION 2

GENERAL CHARACTERISTICS OF THE FRONT PANEL

2.0 Characteristics

Figure 2A illustrates the front panel of the Model 660. Controls are arranged in logical function groups. The majority of front panel controls are momentary contact push-button switches. Many of these switches have a light emitting diode (LED) in their upper lefthand corner. An illuminated LED indicates that the particular control is in an on state. Unless a control's LED is illuminated, its function should assumed to be off . Five of the control pushbuttons (three under CURSOR READS and two under SCALES) have a diagonal / on their face. These controls provide either/or choices.

The instrument has a front panel provision to accept two analog data signals and, optionally, an externally generated sample rate. The results of analyses are presented on the built-in cathode ray tube (CRT) located in the upper lefthand corner of the front panel. Additionally, analog and digital signals are provided on the rear panel, Figure 2B, for hard-copy of screen images as well as external digital communication.

• TO REMOVE REAR PANEL

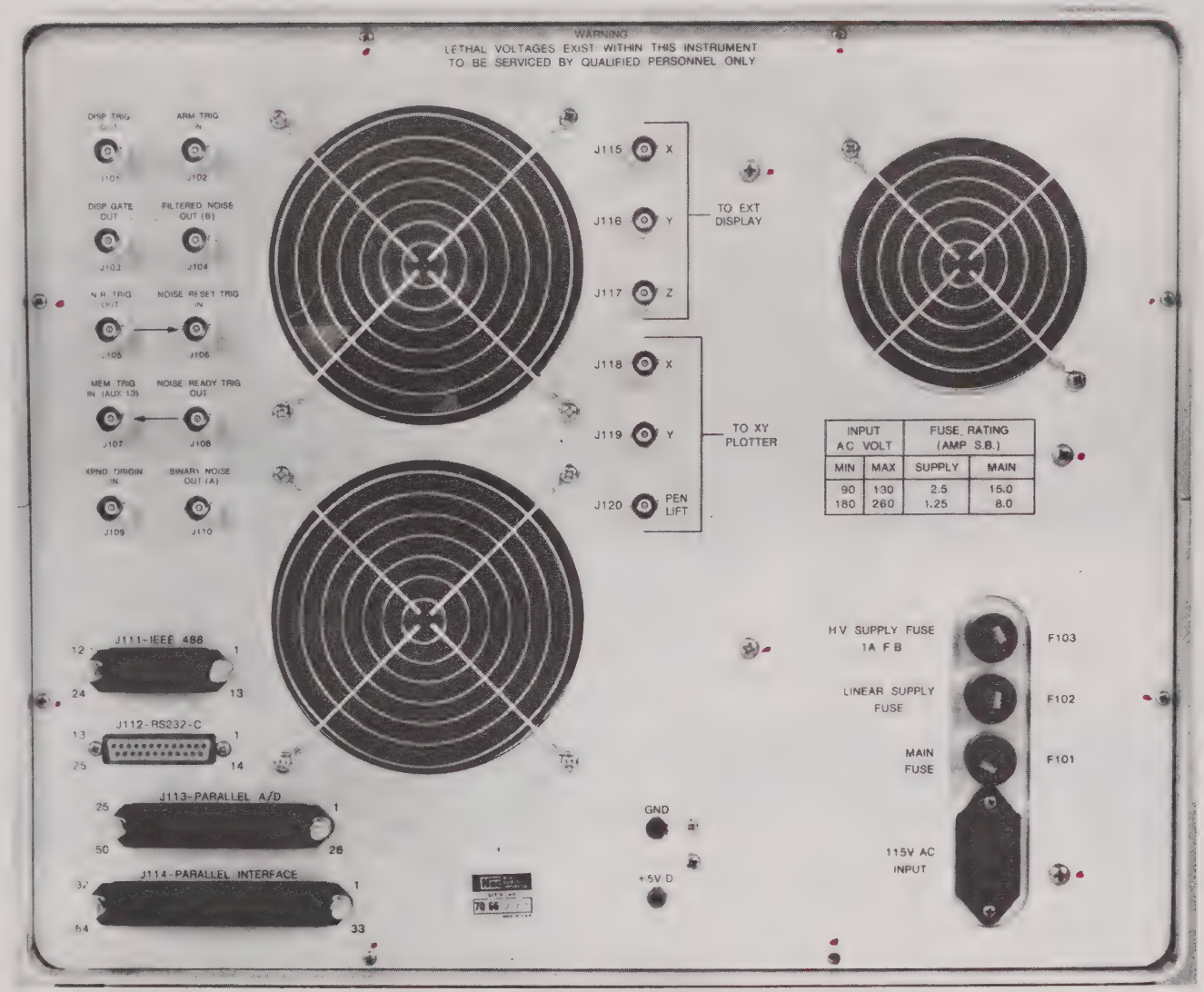


FIGURE 2B

With line power applied to the rear panel line cord mating receptacle, the instrument may be turned on by depressing the POWER pushbutton in the upper righthand corner of the instrument. When power is initially applied, the instrument "wakes up" in dual channel processing mode, displaying the instantaneous spectra (with log amplitude vs. linear frequency) of the applied input signals. Depressing the SYS RESET pushbutton, adjacent to the POWER switch, will also cause the instrument to return to its "wake up" state. See Section 17.

Turning the power off or initiating a system reset will cause the instrument to clear all stored data. The instrument will retain, however, up to ten previously defined operating states. This storage is restricted to the control settings only.

SECTION 3

GENERAL CHARACTERISTICS OF THE DISPLAY

3.0 Display Characteristics

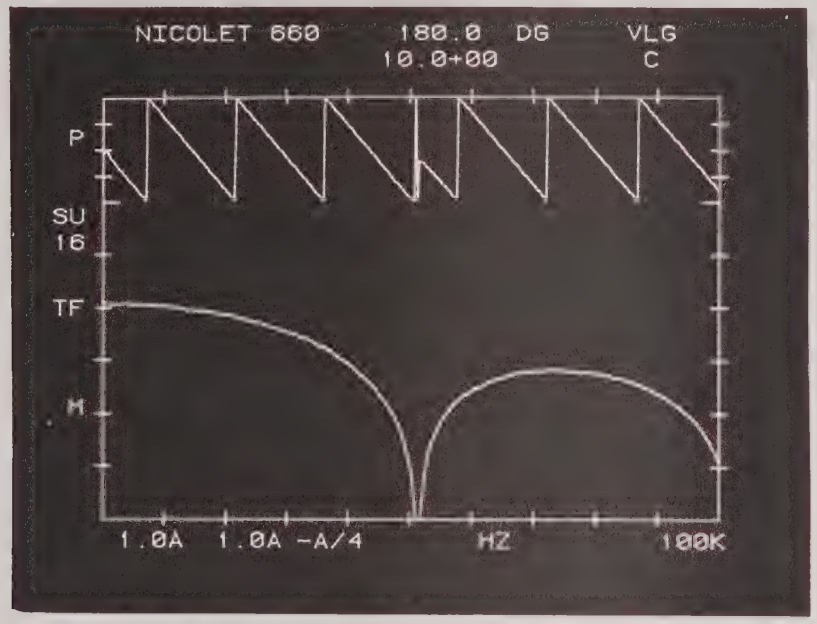


FIG. 3

Figure 3 illustrates a typical CRT display. The display consists of data reflecting the results of computation as well as an electronically drawn graticule and annotation reflecting the control status of the instrument. Additionally, the CRT serves as a communication interface between the user and the instrument.

The annotation NICOLET 660 appearing in the upper lefthand corner of the display, is in the instrument's entry/recall field. This section of the display is reserved for messages defining the status of the instrument. For example, whenever the instrument is operating in its "wake up" state,

the annotation NICOLET 660 will appear. Three types of information may appear in the entry/recall field:

1. A summary of the most recent transaction conducted through the calculation control group, or
2. A status message informing the user that an operation has been accomplished or informs the user a particular mode of operation is ongoing, or
3. An error message indicating that the user has attempted to perform an operation out side the context of the instrument's repertoire.

Error messages require corrective action on the part of the operator. These messages are in English and instruct the operator as to the corrective action that is necessary. Error messages are distinguishable from status messages and calculation control summaries because they are presented in inverted characters. That is, all error messages are presented in dark characters against a white background while normal status messages are presented in light characters against a dark background.

The instrument has the capability to reproduce the CRT display, in its entirety, on the optional Model 134A Digital Plotter. This same information can also be recorded using the Model 61 Polaroid Recording Adapter. Additional facility is provided to record the data traces only, on an ordinary analog X-Y plotter. Digital interface capabilities provide for the transmission of the information content, but not the graphic format, to external digital devices.

SECTION 4

OPERATING MODE CONTROL GROUP

4.0 Operating MODE

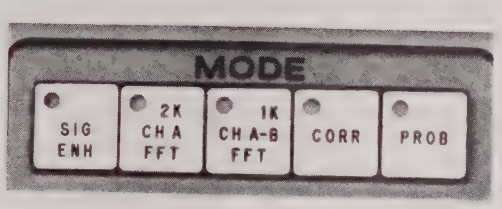


FIGURE 4

Figure 4 illustrates the operating mode control group located in the top central portion of the front panel. One of five operating modes may be selected by depressing the appropriate pushbutton control. The selected mode of operation is indicated by its associated LED. Selection of an operating mode determines what functions of the input data are computed.

Reading from left to right, the five operating modes are:

1. SIG ENH, Signal enhancement; providing dual channel, 1024 point resolution, time domain analysis.

2. 2K CHA FFT, 2K Channel A Fast Fourier Transform; providing single channel, data from channel A, time and frequency domain analyses. Frequency functions are provided with a resolution of 800 points while time domain functions are provided with a resolution of 2048 (2K) points.

3. 1K CHA-B FFT, 1K Channel AB Dual Fast Fourier Transform; providing dual channel processing in the time and frequency domains. Frequency domain functions are presented with a resolution of 400 points while time domain functions are presented with a resolution of 1024 (1K) points.

4. CORR, Correlation; providing the same general characteristics as the preceding mode with parameters optimized for auto and cross correlation functions.

5. PROB, Probability; providing dual channel analysis in the time and amplitude domains. Amplitude domain functions are presented with a resolution of 127 points while time domain functions are presented with a resolution of 1024 points.

The functions available in each mode are summarized in Table 4-1. Note that correlation functions are available

TABLE 4-1

AVAILABLE FUNCTIONS BY MODE

FUNCTION	MODE				
	Signal Enhancement	2K FFT (CH A Only)	1K FFT (A, B Dual)	Correlation	Probability
Input Time History	A & B	A	A & B	A & B	A & B
Averaged Time History	A & B				
Impulse Response			✓	NR	
Instant Spectrum	(1)	A	A & B	NR	
RMS Spectrum		A	A & B	NR	
Power Spectrum		A	A & B	NR	
Coherent Output Power Spectrum			✓	NR	
Cross Spectrum			✓	NR	
Transfer Function			✓	NR	
Transmissibility Function			✓	NR	
Coherence Function			✓	NR	
Auto Correlation Function			NR	✓	
Cross Correlation Function			NR	✓	
Probability Density Function					A & B
Cumulative Distribution Function					A & B

A — Single Channel Function Available for Channel A Only
A&B — Single Channel Function Available for Channels A and B
✓ — Cross Property Function Available and Optimized
NR — Cross Property Function Available but Not Recommended
(1) — In this mode only, it provides instant spectra of averaged time histories for both A and B channels.

in the 1K channel AB fast Fourier transform mode but that certain processing parameters may not be optimized for these functions. Conversely, in the correlation mode, all of the 1K channel AB FFT functions are also available. In the correlation mode these functions will be compromised. Hence, correlation mode should be selected if correlation functions are of fundamental interest; 1K channel AB FFT mode should be selected for all other cross property functions.

In the "wake up" state the instrument automatically selects the 1K channel AB FFT mode.

SECTION 5

FREQUENCY RANGE CONTROLS

5.0 FREQUENCY RANGE Controls

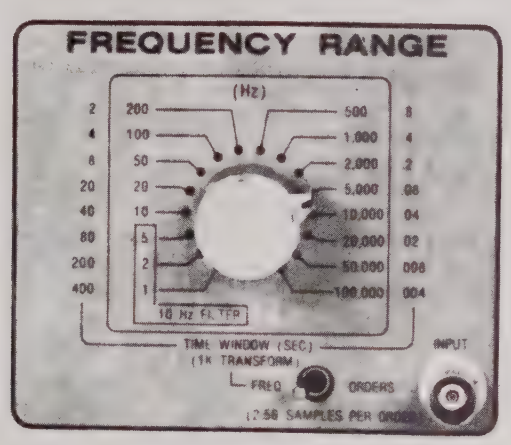


FIGURE 5A

Figure 5A illustrates the frequency range control group located in the lower central portion of the front panel. Sixteen frequency ranges between 1 Hz and 100 kHz full scale may be selected in a 1, 2, 5 progression. The selected frequency range is the maximum frequency analyzed in FFT processing modes. The outer annotation surrounding the frequency range switch specifies the duration of the memory load period in seconds associated with a single 1024 point transform. The memory load period in 2K FFT mode is twice this duration.

The setting of the frequency range switch determines the sample rate at which the data is acquired, for internal

sampling, and the bandwidth of the anti-aliasing filters preceding the instrument's analog-to-digital converters. Note that selecting the 1, 2, or 5 Hz range activates the instrument's 10 Hz anti-aliasing filter. When operating on these frequency ranges, external anti-aliasing filters should be provided.

The internal sample rate provided by the analyzer is always equal to 2.56 times the selected frequency range. The internal sample rate is provided automatically when the FREQ/ORDERS switch is in the FREQ position. Placing this switch in the ORDERS position allows the instrument to operate with an external sample rate. The external sample rate is applied through the front panel INPUT BNC connector. When operating with an external sample rate, the maximum frequency analyzed is equal to the sample rate divided by 2.56. The external sample rate does not have to be constant in frequency. This facility is particularly useful in the analysis of rotating machineries.

When operating with the FREQ/ORDERS switch in the ORDERS position, the frequency range control determines the anti-aliasing filter preceding the analog-to-digital converters. If external tracking anti-aliasing filtering is provided, the frequency range selector should be placed in the 100 kHz position.

Settings of the FREQUENCY RANGE and FREQ/ORDERS controls are annotated on the CRT display. When frequency functions are selected, Figure 5B, the axis label HZ indicates that the FREQ/ORDERS switch is in the FREQ position. Placing this switch in the ORDERS position changes the axis label to OR. The setting of the frequency range switch is always annotated on the extreme righthand edge of the axis. When external sampling is used, this annotation indicates the anti-aliasing filter used.

Figure 5C illustrates the CRT display of a time function. The horizontal axis label SEC indicates that the horizontal axis is in seconds. When the FREQ/ORDERS switch is in the ORDERS position, this label changes to SMP indicating that the horizontal readout is in samples. The time period of the display (memory load period) is indicated at the extreme right of the horizontal axis. When external sampling is selected, this annotation reverts to the setting of the FREQUENCY RANGE switch, indicating the bandwidth of the anti-aliasing filters in Hz.

Figure 5D illustrates a cumulative probability distribution function amplitude domain display. The setting of the FREQUENCY RANGE switch is annotated in the upper righthand corner.

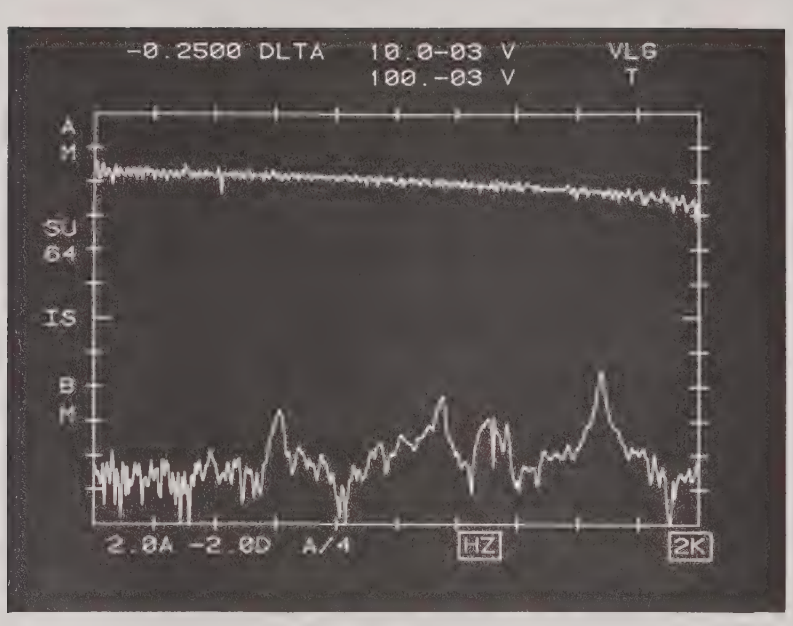


FIGURE 5B

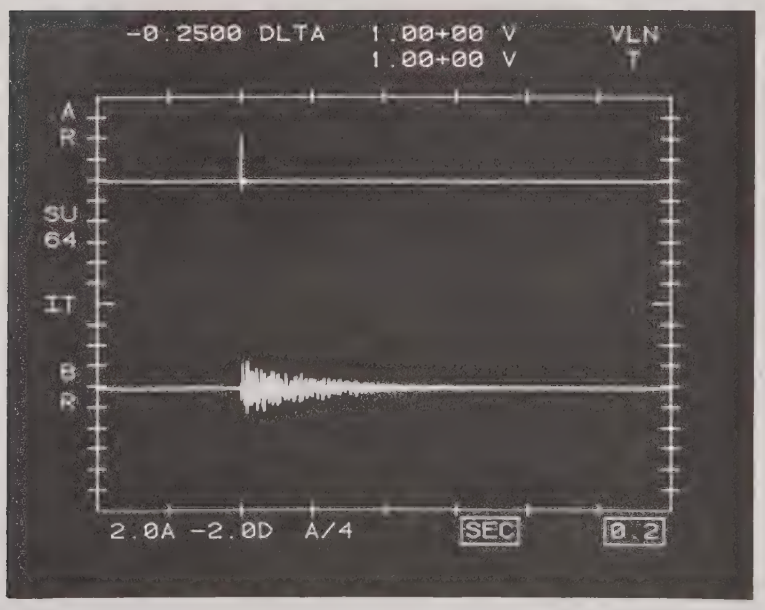


FIGURE 5C

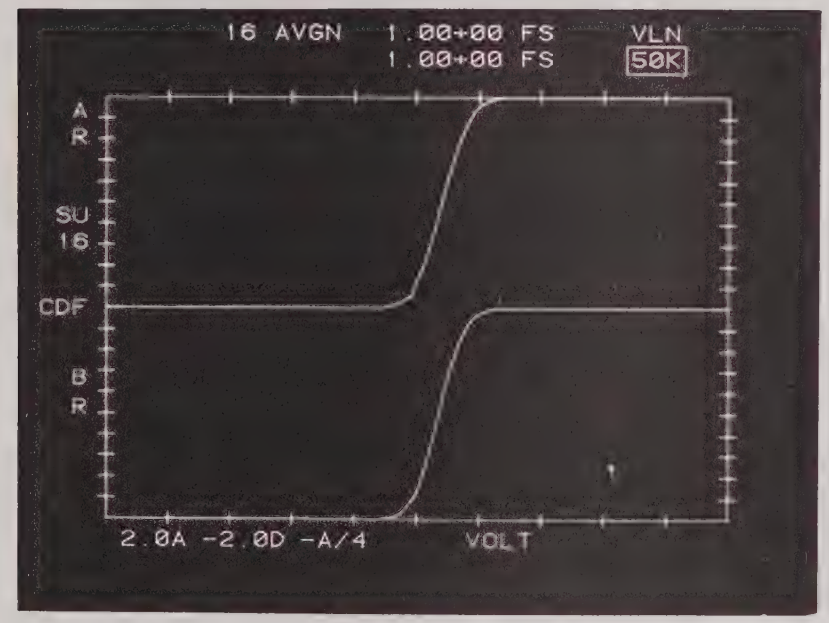


FIGURE 5D

SECTION 6

INPUT CHANNEL A AND INPUT CHANNEL B CONTROL GROUPS

6.0 INPUT Control Groups

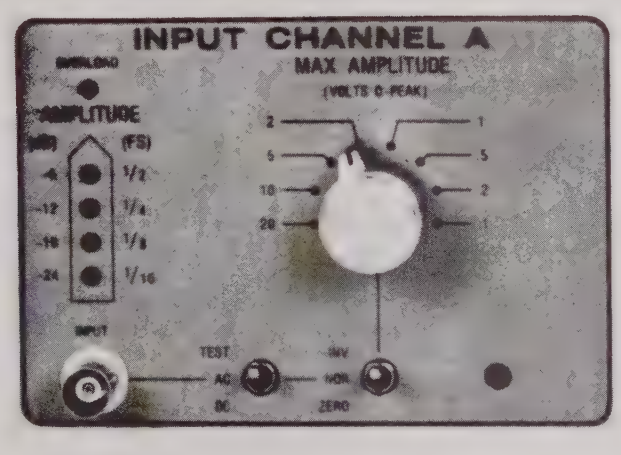


FIGURE 6A

Figure 6A illustrates one of two identical input control groups located in the lower corners of the front panel. The data signal to be analyzed is applied to the input BNC connector. AC or DC coupling may be selected, or a test signal may be substituted for the input. When AC coupling is selected, the 3 dB point is located at less than 1 Hz. When the test signal is selected, a periodic signal at 64% of the selected analysis range is applied.

The signal may be analyzed as applied (normal) or it may be inverted. Inverting the signal changes the sign

of its amplitude. Alternatively, an input ZERO may be selected. Selecting an input ZERO provides a digital zero code output from the analog-to-digital converter but does not affect the electrical characteristics of the input BNC.

The MAX AMPLITUDE selector provides eight choices of input sensitivity from .1 volt to 20 volts (0-PEAK) in a 1, 2, 5 progression. Proper setting of the MAX AMPLITUDE control is achieved by viewing the AMPLITUDE LEDs. Proper setting of the MAX AMPLITUDE control is achieved when the OVERLOAD LED is off and as many of the remaining LEDs as possible (signal dependent) are on.

Settings of the input controls are annotated on the CRT image as shown in Figure 6B. These annotations always appear in the lower lefthand corner of the display. Annotation for channel A (left channel) appears on the left side. Annotation for channel B appears on the right side. If the signal has been inverted a minus sign is displayed. The selected MAX AMPLITUDE voltage is displayed. This number reads 0.0 if a digital "0" has been applied. AC coupling is indicated by the letter A, DC coupling by the letter D, and the test signal by the letter T.

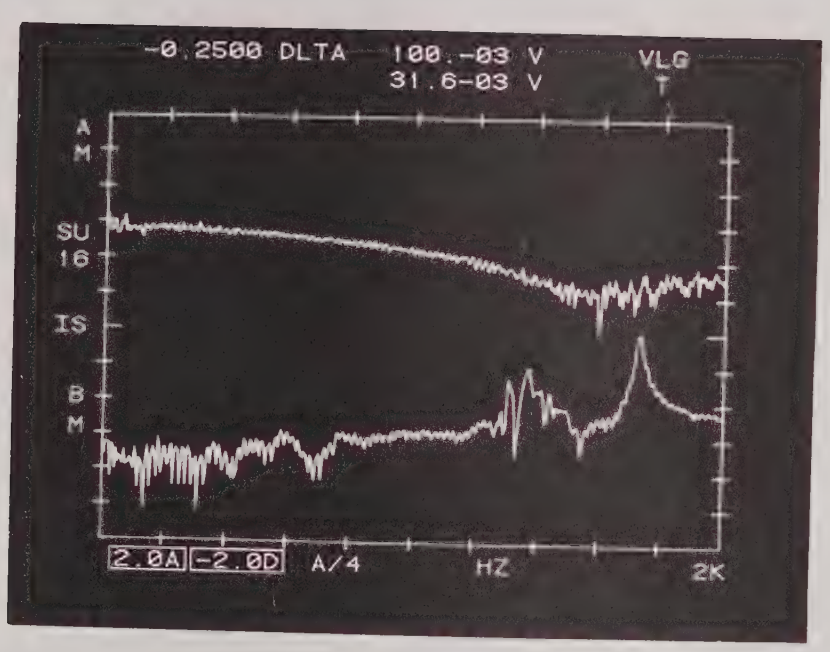


FIGURE 6B

SECTION 7

CAPTURE CONTROLS

7.0 CAPTURE CONTROL Group

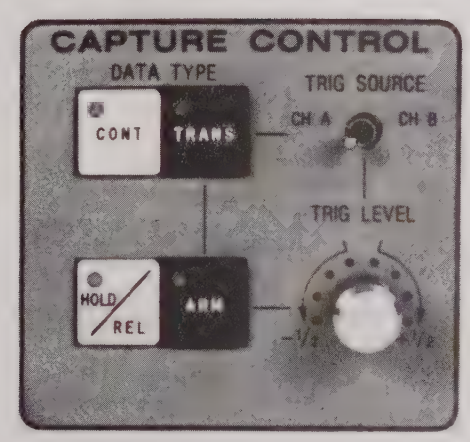


FIGURE 7A

Figure 7A illustrates the capture control group located in the lower central portion of the front panel. The DATA TYPE controls allow selection of processing of continuous or transient signals. When the CONT button is depressed, a Hanning squared, cosine^4, window is applied to the data. When the TRANS button is depressed, the data is analyzed using a uniform window. Hence, the data may be weighted or unweighted.

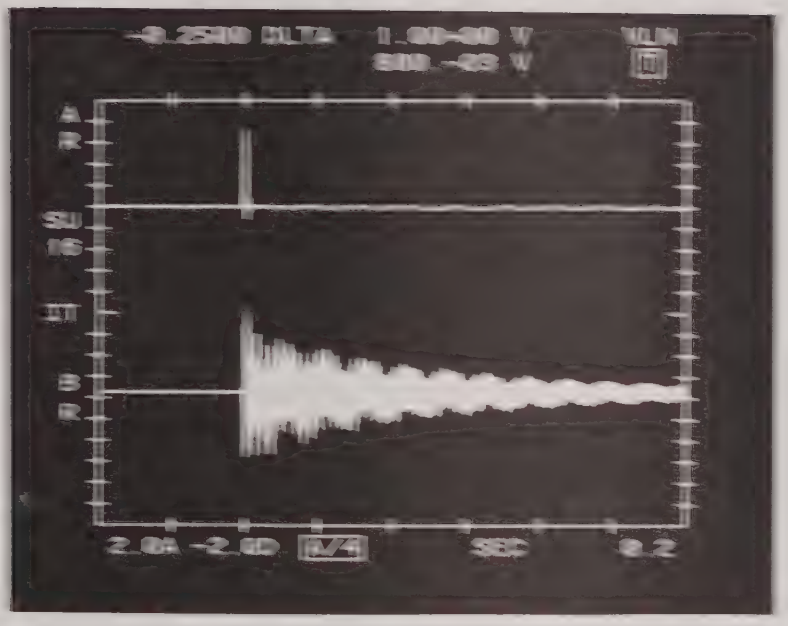
The instrument may be caused to hold its current input data window by depressing the manual HOLD button. When

this button is depressed, its associated LED lights. Depressing the button a second time causes the data to be released. This button permits freezing portions of the input signals for further examination.

Alternatively, the instrument may be programmed to automatically go into hold. This is accomplished by selecting either channel A or channel B to be the trigger source. The signal level which causes the hold state may be selected using the TRIG LEVEL rotary switch. This selector permits levels from $\pm$ 1/16 to $\pm$ 1/2 of the selected MAX AMPLITUDE to be chosen.

Depressing the ARM button, causing its LED to light, arms the analyzer to anticipate the transient capture conditions. When these conditions are met, the system automatically goes into a hold state.

Figure 7B illustrates the display annotation reflecting the status of the capture controls. The circled T indicates that the transient data type has been selected. Continuous data type is indicated with a C. When operating in correlation mode a special window is provided regardless of the settings of the data type controls. This window is uniform over the first half of the time window and zero over the second half of the time window. The correlation window is indicated by the annotation ZC for zeroed correlation


SEC

window.

Annotation below the display indicates the status of the TRIG SOURCE and TRIG LEVEL controls. Absence of a sign is a positive trigger level. A minus sign is used for negative levels. The letter A or B indicates which channel is the trigger source. The suffix, / 4, indicates the trigger level is 1/4 of the selected maximum amplitude.

SECTION 8

AVERAGING CONTROL GROUP

8.0 AVERAGE Control Group

FIGURE 8A

Figure 8A illustrates the averaging control group. One of nine averaging modes may be selected. These are:

1. SUM, Summation
2. DIFF, Difference
3. EX, Exponential
4. PEAK, Peak
5. SWEEP, Sweep
6. TRANS SUM, Transient Summation
7. TRANS DIFF, Transient Difference
8. TRANS EX, Transient Exponential
9. TRANS PEAK, Transient Peak

When operating in signal enhancement, the instrument performs ensemble averaging for channel A and B data in the time domain. When operating in the 2K channel A FFT mode the instrument performs ensemble averaging of power, rather than voltage, spectra of data applied to channel A. When operating in either the 1K channel A-B FFT or the correlation modes, the instrument performs ensemble averaging of the power spectra for both channel A and channel B as well as ensemble averaging of the real and imaginary components of the cross spectrum between channels A and B. When operating in the probability mode, the instrument performs accumulation of occurrence and amplitude of the data applied to both channels A and B.

When summation averaging is selected, the averager section performs summation averaging over N preset spectra or time windows. The number N is entered through the calculation control group discussed in Section 16. In summation averaging, each constituent in the average is weighted equally. When averaged functions are displayed, their amplitude is continuously normalized based upon the number of

constituents currently averaged. Summation averaging may be conducted on statistically independent time windows, non-redundant data, or upon sequential time records that overlap by no more than 50% (50% redundancy). Averaging redundancy is determined by entry through the calculation control group.

Difference averaging is very similar to summation averaging except that each constituent is subtracted, rather than added, from the average. Normally, difference averaging is performed following a summation average. Difference averaging permits subtracting a noise background from experimental measurements. As implemented in the 660, difference averaging may be performed either before or after the summation average with the same effect.

Exponential averaging is a moving average in time. In this mode of averaging, the most recent information is weighted heavily while the oldest history is weighted lightly. The weighting function between these extremes is time exponential, hence the name. Exponential averaging is equivalent to placing a resistor-capacitor (RC) filter or leaky integrator on each point in the spectrum.

Peak averaging is not truly an average. It provides a one time statistic. In peak averaging, the largest event that occurs at any frequency or time position is stored in

the averager memory. Peak averaging provides the "worst case" spectrum or time record. When peak averaging is selected, every spectrum computed by the 660 is tested to see if the value within it has exceeded a previously stored value. This processing is, therefore, performed at maximum redundancy.

Sweep averaging is specifically designed for the analysis of systems excited by swept sine. Sweep averaging is summation averaging conducted at maximum redundancy. That is, summation averaging is performed upon every spectrum computed by the 660, rather than waiting for the required redundancy conditions.

Transient averaging is a special mode of averaging optimized for the analysis of transient signals. When these averaging modes are selected, the instrument automatically re-arms itself after each transient acquisition. If an acquired transient caused an overload of the instrument's analog-to-digital converters, it is not averaged. The instrument only averages spectra or time histories resulting from transient inputs that satisfied the trigger conditions but did not overload the input of the instrument. This averaging mode is particularly useful when examining structures using impulsive techniques as discussed in Appendices D, E, and F. The transient mode is available when using summation, difference,

exponential or peak averaging.

An averaging process is initiated by depressing the START pushbutton. The LED on the START button remains lighted while averaging is being conducted. At the conclusion of the averaging period, this light turns off. Depressing the START button erases any previously averaged data and starts the averaging count from one.

Normally, averaging is conducted until the preset averager N is reached. The process may be manually stopped or interrupted by depressing the STOP pushbutton. To resume the averaging process, depress the CONT, continue, pushbutton. Depressing the CONT pushbutton does not erase prior information nor does it reset the averaging counter. The CONT pushbutton may be used to average data in excess of the preset N without incurring any scaling errors.

Figure 8B illustrates CRT annotation describing the status of the averaging process. The present averaging conditions are specified by a 2 or 3 letter code for averaging mode directly above the preset averager N, i.e., SU 100 shown here. The averaging codes are:

1. SU for Summation
2. DF for Difference
3. EX for Exponential

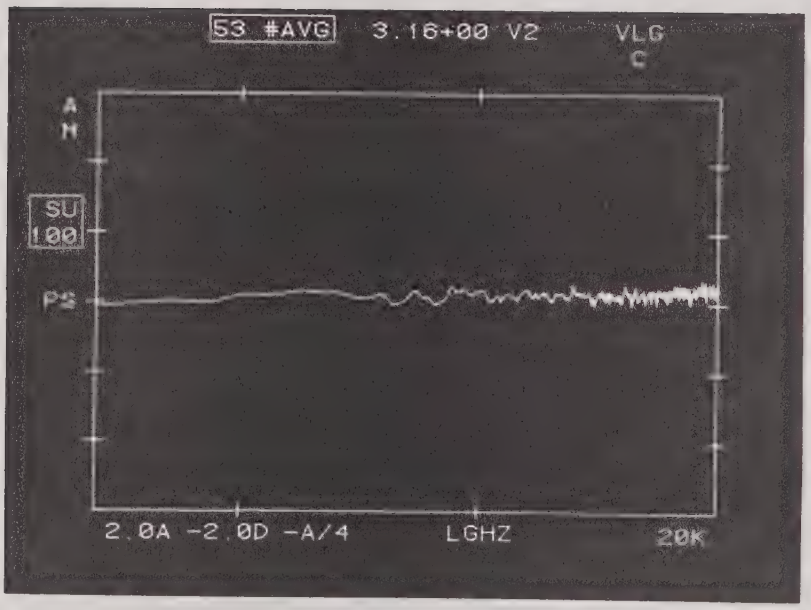


FIGURE 8B

4. PK for Peak

5. SW for Sweep

6. TSU for Transient Summation

7. TDF for Transient Difference

8. TEX for Transient Exponential

9. TPK for Transient Peak

Additionally, the calculation control group permits displaying the running averager count as the process proceeds. The annotation 53 # AVG, shown here indicates that 53 of the preset 100 averages have been acquired at this instant in time. Table 8-1 summarizes the system's averaging modes.

In the "wake up" state the averager is preset to summation averaging mode with N equal to 16.

TABLE 8-1

VALID AVERAGING MODES

OPERATING MODE / AVG MODE	SIG ENH	2K CH A FFT	1K CH A-B FFT	CORR	PROB
1. SUMMATION	X	X	X	X	X
2. DIFFERENCE	X	X	X	X	
3. EXPONENTIAL	X	X	X	X	
4. PEAK		X	X	X	
5. SWEEP		X	X	X	
6. TRANSIENT SUMMATION	X	X	X	X	X
7. TRANSIENT DIFFERENCE	X	X	X	X	
8. TRANSIENT EXPONENTIAL	X	X	X	X	
9. TRANSIENT PEAK		X	X	X	

SECTION 9

FUNCTION CONTROL GROUP

9.0 FUNCTION Control Group

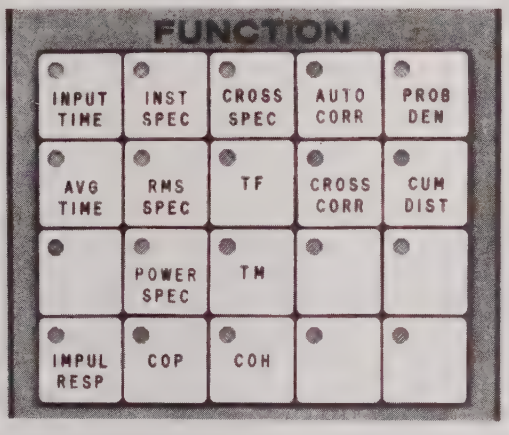


FIGURE 9A

Figure 9A illustrates the function control group. Depressing a button in this group causes the display of a selected function. If the selected function is not available in the selected operating mode, the error message FUNCTION N/A, for function not available, will be seen in the entry/recall field. Functions available by mode are listed in Table 4-1.

All functions with the exception of INPUT TIME, input time histories, and INST SPEC, instantaneous spectra, require averaging for their computation. Hence, the averaging process must be started before meaningful displays of most

functions will be obtained. Any function may be viewed while the averaging is in process.

The selected function is denoted on the CRT display by a function label. In Figure 9B, the annotation PS indicates that a power spectrum is being displayed.

Table 9-1 summarizes the available functions, their front panel abbreviations in the function control group, their CRT function labels, and modifications to these labels indicating the effects of controls described in Sections 12 and 15. Definitions of the functions are provided in Appendices G, H and I. Table 9-2 gives the available formats for the different functions.

In the "wake up" state, instantaneous spectrum is the selected function.

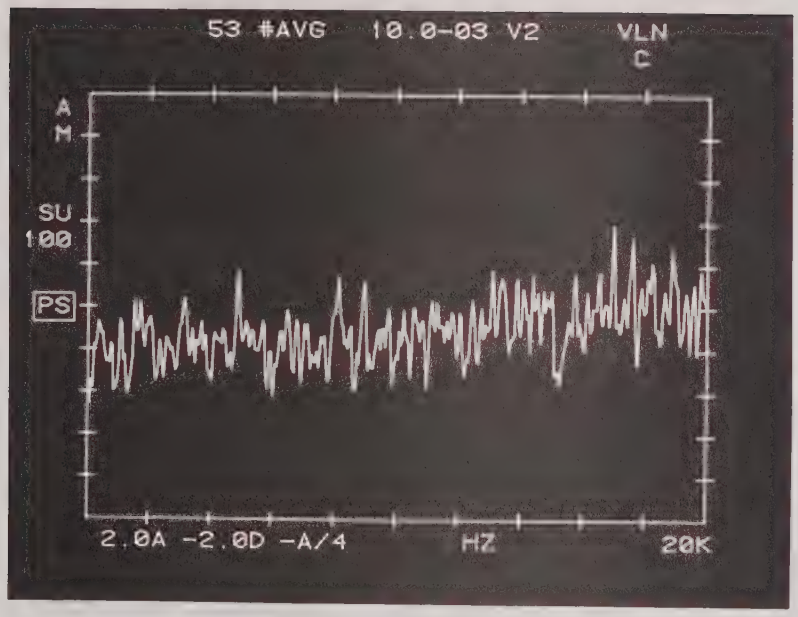


FIGURE 9B

FUNCTION	PANEL ABBREVIATION	CRT LABEL	MODIFIED LABEL*	AVERAGING REQUIRED	DATA TYPE
INPUT TIME HISTORY	INPUT TIME	IT	-	NO	REAL
AVERAGED TIME HISTORY	AVG TIME	AT	-	YES	REAL
IMPULSE RESPONSE	IMPUL RES	IR	EIR	YES	REAL
INSTANT SPECTRUM	INST SPEC	IS	ISD	NO	COMPLEX
RMS SPECTRUM	RMS SPEC	RS	RSD	YES	MAGNITUDE
POWER SPECTRUM	POWER SPEC	PS	PSD	YES	MAGNITUDE
COHERENT OUTPUT POWER SPECTRUM	COP	CP	CPD	YES	MAGNITUDE
CROSS SPECTRUM	CROSS SPEC	CS	CSD	YES	COMPLEX
TRANSFER FUNCTION	TF	TF	ETF	YES	COMPLEX
TRANSMISSIBILITY FUNCTION	TM	TM	ETM	YES	MAGNITUDE
COHERENCE FUNCTION	COH	COH	-	YES	REAL
AUTO CORRELATION FUNCTION	AUTO CORR	ACF	-	YES	REAL
CROSS CORRELATION FUNCTION	CROSS CORR	CCF	-	YES	REAL
PROBABILITY DENSITY FUNCTION	PROB DEN	PDF	-	YES	REAL
CUMULATIVE DISTRIBUTION FUNCTION	CUM DIST	CDF	-	YES	REAL

* "E" Prefix Denotes Equalization, i.e. Equalized Impulse Response = EIR
"D" Suffix Denotes Density (per Hz) Normalization, i.e. Power Spectral Density = PSD

TABLE 9-1

FUNCTION LABELS

TABLE 9-2

AVAILABLE DISPLAY FORMATS

	INPUT TIME	AVG TIME	IMPULSE RESP	INST SPEC	RMS SPEC	POWER SPEC	COP	CROSS SPEC	TF	TM	COH	AUTO CORR	CROSS CORR	PROB DEN	CUM DIST
A REAL	x	x		x								x		x	x
A IMAG				x											
A R/I				x											
A MAG				x	x	x									
A PHASE				x											
A PH/MAG				x											
A NYQ				x											
B REAL	x	x		*								x		x	x
B IMAG				*											
B R/I				*											
B MAG				*	*	*									
B PHASE				*											
B PH/MAG				*											
B NYQ				*											
AB REAL	x	x		*								x		x	x
AB IMAG				*											
AB R/I															
AB MAG				*	*	*									
AB PHASE				*											
AB PH/MAG															
AB NYQ	x	x													
XP REAL			x					x	x		x	x	x		
XP IMAG								x	x						
XP R/I								x	x						
XP MAG							x	x	x	x					
XP PHASE								x	x						
XP PH/MAG								x	x						
XP NYQ								x	x						

* Not Valid in 2K
x Valid Formats

SECTION 10

DISPLAY (CHANNEL/FORMAT) CONTROLS

10. DISPLAY Controls

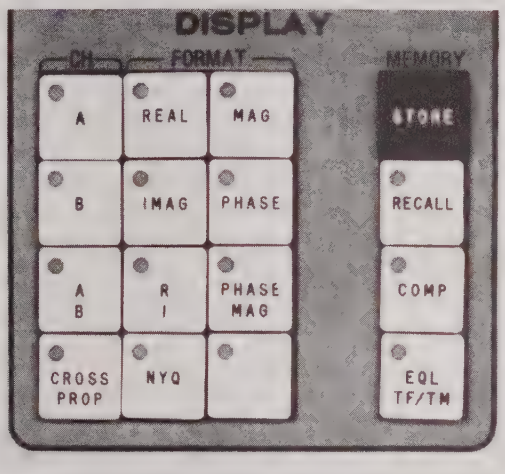


FIGURE 10A

Figure 10A illustrates the display control group. Controls are provided to select a channel (CH) and a format (FORMAT).

The CH pushbuttons select the channel from which the selected function will be displayed. For single channel functions only CH A selection is valid. For dual channel functions, choices of channel A or channel B or channels A & B are provided. The available selections are listed in Table 4-1.

The FORMAT pushbuttons determine how the selected single channel or cross property function will be displayed.

The function to be displayed may have one of three format types, namely: complex, magnitude or real. The format types are specified in Table 9-1.

A single complex function contains two pieces of information. In retangular coordinates these are called the real and imaginary components, while in polar coordinates, they are called the magnitude and phase components. These terms are discussed in Appendix H.

When a complex function is displayed, its real, imaginary, or both its real and imaginary, RI, may be displayed. Alternatively, the imaginary component may be cross plotted against the real component by selecting a Nyquist display format. Additionally, the same information may be plotted in polar coordinates. Choices of magnitude, phase, or both phase and magnitude may be selected.

For data whose format type is either magnitude or real only those formats are valid, respectively. Refer to Table 9-1 for further clarification.

When a function of either real or magnitude type is selected, the instrument automatically selects the appropriate format. When a complex function is selected, the instrument automatically selects the last manual selection for format.

The 660 has provisions to display one or two traces at a time. This permits the simultaneous display of two real or two magnitude functions or the entire display of a single complex function. Additionally, a single attribute, i.e. magnitude, phase, real, or imaginary, of two complex functions, one from channel A and one from channel B, may be displayed simultaneously. Attempting to display more than two traces or selecting an improper format may result in an entry/recall field error message. These messages are summarized in Appendix B.

The Nyquist display format button normally is used to cross plot a complex function's real and imaginary components. This format may also be selected to cross plot two time histories, either instantaneous or averaged.

Figures 10B, 10C, and 10D illustrate the CRT annotation reflecting the status of the display controls. In Figure 10B, two instantaneous spectra are displayed. The upper trace is the data from channel A while the lower trace is the data from channel B. Both displays are of spectrum magnitude. In Figure 10C a transfer function (cross property function) is displayed. The upper trace is the phase spectrum while the lower trace is the magnitude (ratio) spectrum. In Figure 10D the real (horizontal) and imaginary (vertical)

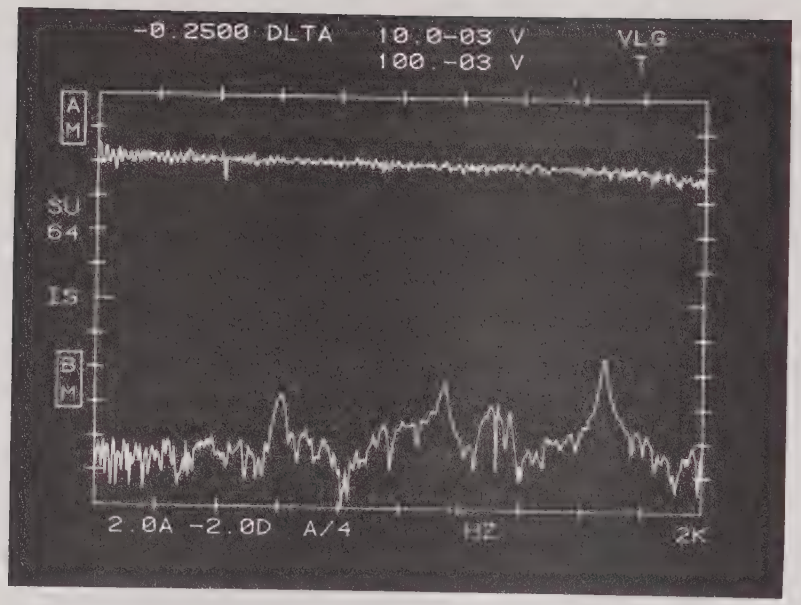


FIGURE 10B

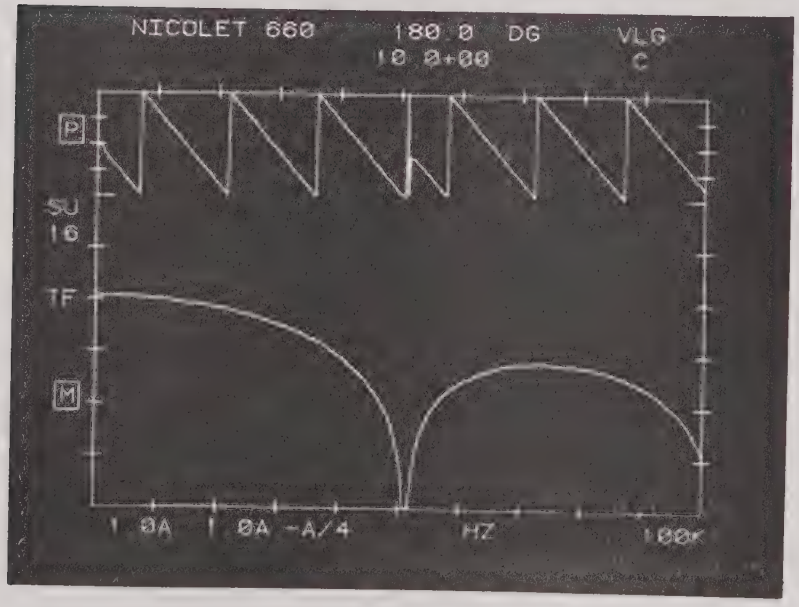


FIGURE 10C

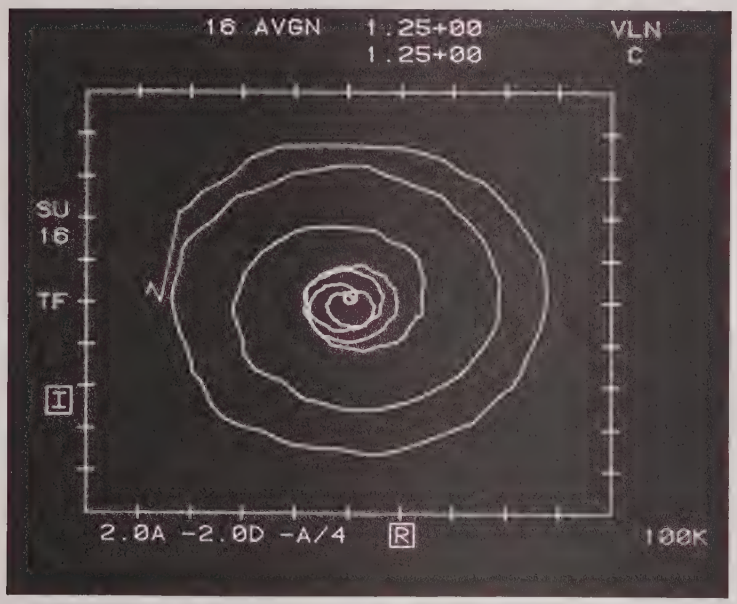


FIGURE 10D

components of a transfer function are cross plotted in Nyquist format.

In the "wake up" state, channel is set to A-B while format is set to magnitude.

SECTION 11

DISPLAY SCALES CONTROLS

11. Display SCALES Controls

FIGURE 11A

Figure 11A illustrates the SCALES controls. These pushbuttons allow alternate selection of linear or logarithmic horizontal and vertical axes for the CRT display. When logarithmic axes are selected, the LEDs are illuminated. When linear axes are selected, the LEDs are off.

The amplitude scales control only affects functions whose data type is complex or magnitude. Real functions are always displayed with linear amplitude.

The lower frequency scales control affects functions of frequency. Time functions are always presented with a linear horizontal axis.

Both the display form and CRT annotation reflect the status of the SCALES controls. Figure 11B illustrates a power spectrum presented with linear amplitude vs. linear frequency. The vertical axis label, VLN, indicates that the vertical axis of this display is linear. When linear displays are drawn, nine tick marks (10% apart) are always drawn for each data trace. The horizontal axis label Hz indicates that the horizontal axis is linear. Again, tick marks are 10% apart.

Figure 11C illustrates the same data presented with logarithmic vertical and horizontal axes. The vertical axis label, VLG, indicates that the vertical axis is logarithmic. The tick marks on logarithmic vertical displays are always 10 dB apart. The display range, or number of 10 dB intervals, may be specified through the calculation control group. The horizontal axis label, LGHz, indicates that the horizontal axis is logarithmic. The tick marks are one decade apart.

In the "wake up" state the amplitude scale is set to logarithmic while the frequency scale is set to linear. The display range is set to 60dB.

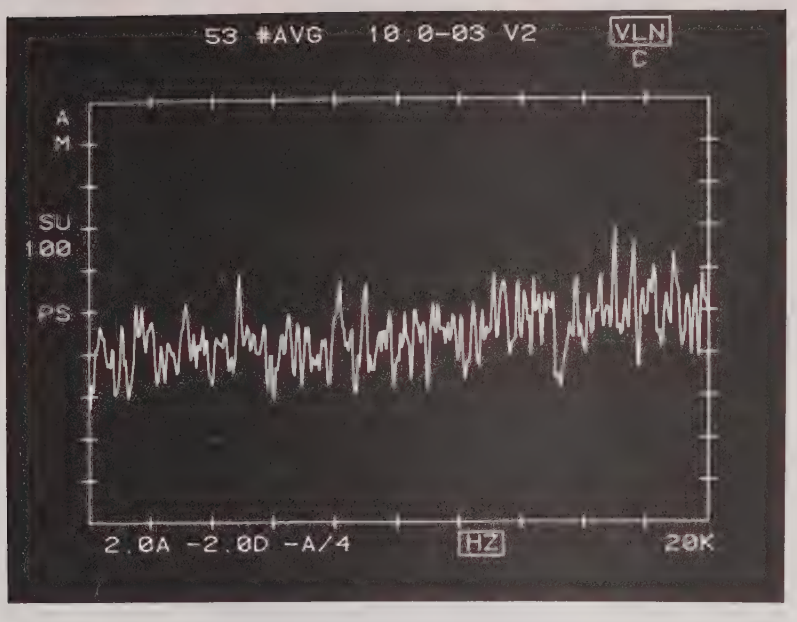


FIGURE 11B

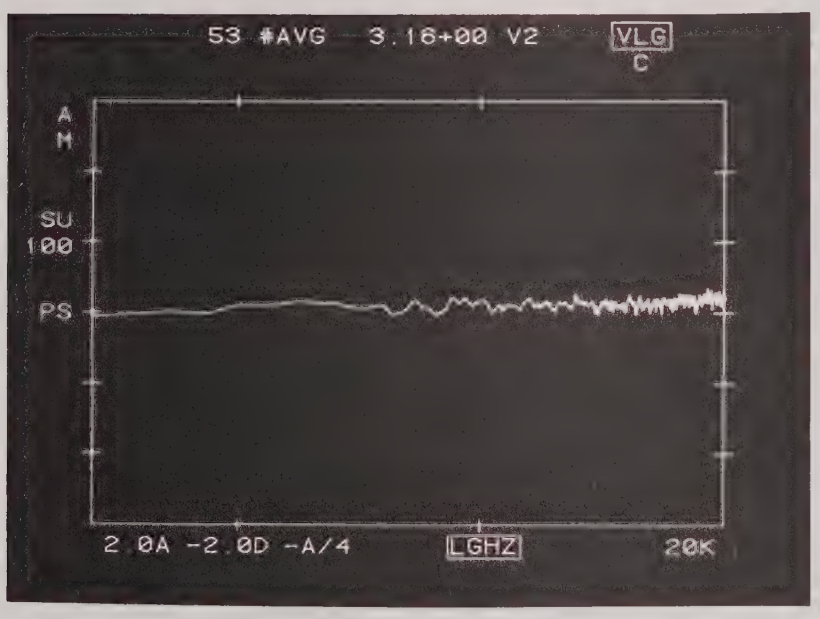


FIGURE 11C

SECTION 12

DISPLAY GAIN AND UNITS CONTROLS

12. GAIN and CURSOR READS

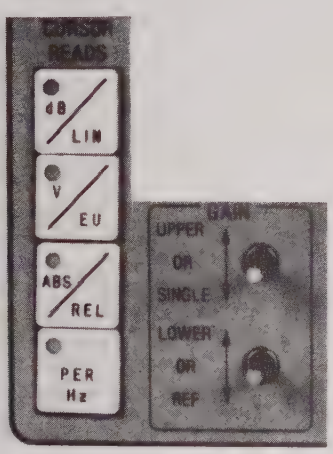


FIGURE 12A

Figure 12A illustrates the controls that select amplitude units and display gain. The pushbuttons grouped under the heading CURSOR READS determine not only the cursor readings but the scaling of the display traces. These four pushbuttons perform alternate action selection.

The ABS/REL switch determines whether the vertical units of the display and cursor readout will be in absolute or relative units. Absolute units may be either voltages as directly measured by the instrument, or engineering units calculated from the voltages readings and engineering unit

scale constants. The engineering unit scale constants are entered through the calculation control group. Relative units are ratiometric measurements between two data traces or between two points on a single data trace. Relative measurements are made from a reference value entered through the calculation control group.

The V/EU switch determines whether the absolute units will be volts or engineering units. Changing the status of either V/EU or the ABS/REL pushbuttons will affect the vertical scaling of the data, depending upon the engineering unit scale constants and relative reference entered, and may require changing the gain of the display.

The dB/LIN switch determines whether the display full scales and the amplitude-at-cursor readouts will be in linear units (volts, engineering units, or relative) or in dB (with respect to one linear unit). This selection is independent of the amplitude scales selection.

The alternate action PER Hz switch controls the application of density normalization to displayed spectra. Depressing the PER Hz pushbutton normalizes the amplitude of a displayed spectrum to 1 Hz bandwidth. Density normalization (per Hz) may be applied to five frequency functions as specified in Table 9-1. When the PER Hz pushbutton is depressed,

the function label is modified by the addition of the suffix D (for density normalization). Density normalization is discussed in Appendix H.

The GAIN cursors determine the height of the displayed traces. When a single trace is displayed, the UPPER gain cursor determines the size of the display. When two traces are displayed simultaneously, the UPPER gain cursor determines the height of the upper trace while the LOWER cursor determines the height of the lower trace.

Three exceptions should be noted:

1. When the coherence function is displayed, the gain cursors are inactive. Because the coherence function is always bounded between zero and one, there is no need for variable display gain. Hence, the bottom of the display is always zero and the top of the display is equal to one when the coherence function is selected.

2. Phase data is always displayed between $\pm$ 180°. 0° is always the center of a phase display. The GAIN cursors do not affect a phase display.

3. When magnitude and phase format is displayed, the UPPER cursor determines the gain of the magnitude display. The LOWER gain is inactive.

The GAIN cursors actually determine the full scale value of the CRT display. The 660 automatically scales the displayed data so that the full scale is a nice number. When the AMPL SCALES button is in the linear position, pressing a GAIN cursor downward steps the full scale through the sequence 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, or 8 times 10 to an integer exponent ($\pm$ 39). Raising the cursor steps the full scale through the same sequence in the opposite direction. Additionally, explicit keyboard entry for a desired full scale may be made through the calculation control group.

When the AMPL SCALES selector is in the logarithmic position, moving the GAIN cursor causes the display full scale to change in 10 dB steps. If the CURSOR READS dB/LIN button is in the linear position, the full scale will be either 3.16 or 10 multiplied by 10 to an integer power. With this selector in the dB position, the full scale will be an integer power of 10 dB with reference to one physical unit of the displayed measurement.

The CRT display annotation reflects the status of the CURSOR READS and GAINs controls as illustrated in Figures 12b, 12c, 12d, and 12e. In Figure 12B a power spectrum is displayed. The full scale amplitude of the display in volts^2 is displayed. The label V2 indicates that volts^2 are being displayed. If an INST or RMS spectrum had been displayed, the label V for volts would have appeared.

In Figure 12C the V/EU selector has been changed from the volts to the engineering unit position. Additionally, the volt per engineering unit scale constants have been recalled to the entry recall field. The physical readout is now in engineering units^2 as indicated by the E2 label. Readout in linear engineering units is denoted by the symbol E. When using relative measurements rather than absolute, the symbols R and R2 are used. In Figure 12D a power spectrum is displayed with logarithmic amplitude. Note that the vertical units are volts^2 (V2) and that the function label PS indicates a power spectrum is being displayed.

Figure 12E shows the same power spectrum after the PER Hz pushbutton was depressed. Note that the function label has been changed from PS to PSD indicating that density normalization has been applied. The units of the full scale are now volts^2/Hz.

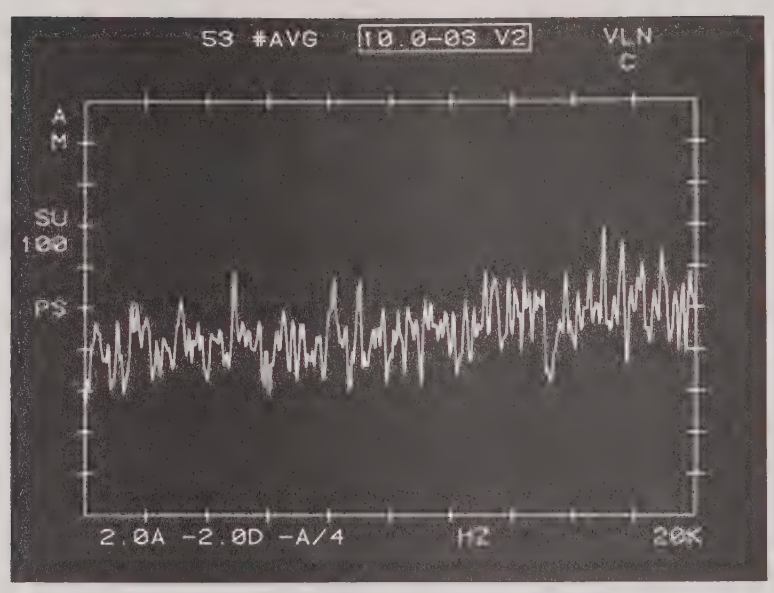


FIGURE 12B

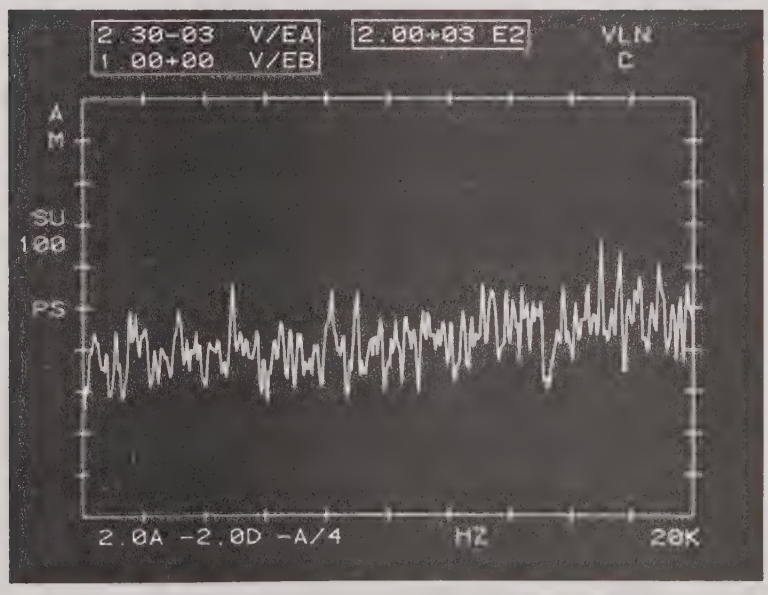


FIGURE 12C

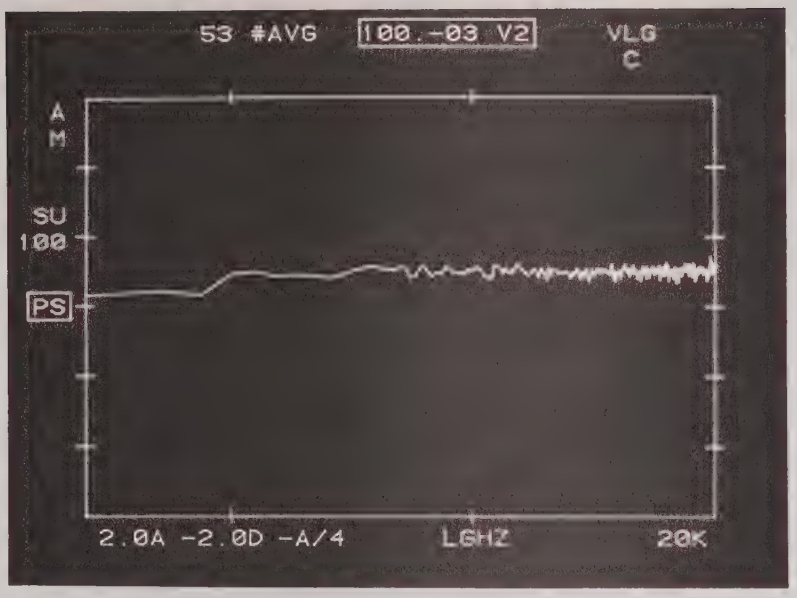


FIGURE 12D

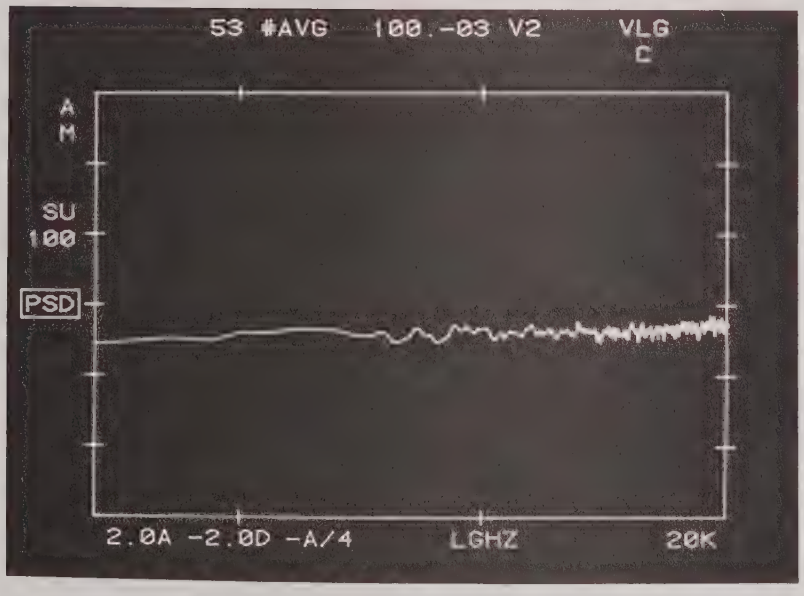


FIGURE 12E

Any combination of the pushbuttons under the CURSOR READS label is valid. Note that the units of amplitude measure are 0 to Peak for functions in the time or amplitude domains and RMS for functions in the frequency domain.

In the "wake up" state the CURSOR READS pushbuttons are initialized to linear, volts, absolute and per Hz off. Additionaly, the GAIN controls are initialized to match the settings of the maximum amplitude controls.

SECTION 13

CURSOR CONTROLS

13. CURSOR POSITION, CURSOR TYPE, and FREQ READS

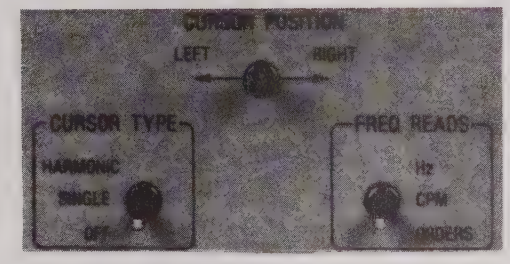


FIGURE 13A

Figure 13A illustrates the cursor control group. The CURSOR TYPE switch permits turning the cursor off, providing a single cursor or a harmonic cursor. Selecting HARMONIC cursor type provides a single line intersecting the data with additional intensified dots at harmonics of the cursor position.

When the cursor is turned on, the CURSOR POSITION switch permits positioning the cursor left or right. Touching this control once advances the cursor one location. Holding this control to the left or right causes the cursor to slew. Alternatively, the cursor position may be explicitly selected

through the calculation control group.

The FREQ READS selector affects the readout of horizontal units. Normally, horizontal readout is in Hz for frequency functions, is in seconds for time functions, and is in voltage or engineering units for amplitude functions. The FREQ READS switch only affects functions of frequency or time. In the Hz position, units of Hz and seconds are used. In the CPM position, units of cycles per minute are displayed in frequency, and fractions of time windows (specified by the frequency range) are displayed in time. In the ORDERS position, orders (normalized frequency) are used for frequency and units of cycles (inverse orders) are used for time. The order reference is established through the calculation control group.

The horizontal cursor readout is effected not only by the FREQ READS selector but also by the FREQ/ORDERS switch in the frequency range control group. This interaction is summarized in Table 13-1.

Figure 13B illustrates a CRT display with the cursor turned on. The cursor takes the form of a vertical line intersecting the data trace or traces. The horizontal position of the cursor is read out directly below the entry recall field.

TABLE 13-1

HORIZONTAL READOUT AT CURSOR

FREQUENCY READS SETTING	DOMAIN OF DISPLAYED FUNCTION		
	TIME	FREQUENCY	AMPLITUDE (PROBABILITY)
HZ	SECONDS (#)	HZ (#)	VOLT OR EU (SAME)
CPM	FS (FS)	CPM (FS)	VOLT OR EU (SAME)
ORDERS	CYC (CYC)	ORD (ORD)	VOLT OR EU (SAME)

HZ = Hertz (Cycles/Second)

CPM = Cycle/Minute

ORD = Order (Frequency Ratio)

FS = Fraction of Full Scale (Frequency Range or Time Window)

CYC = Cycles ($Order^{-1}$)

\# = Point or Cell Number

() = Indicates FREQ/ORDERS Switch in ORDERS Position

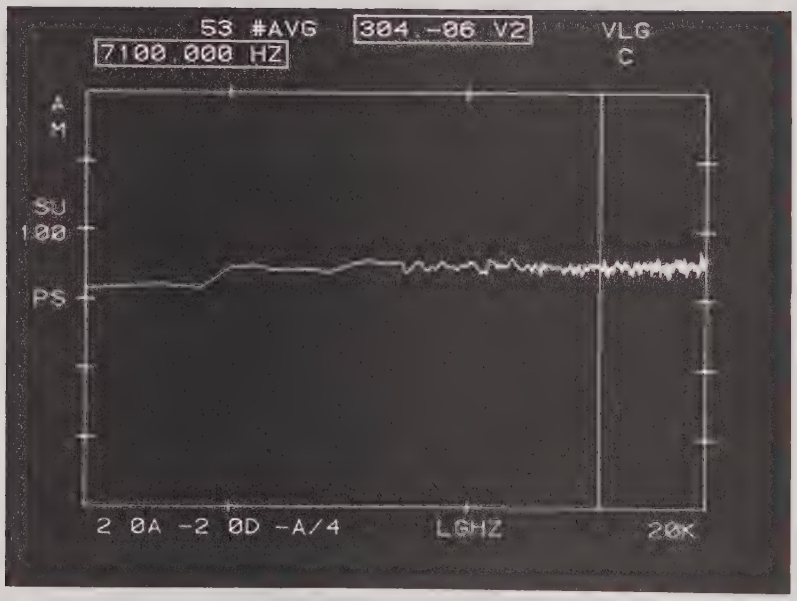


FIGURE 13B

In this example, the frequency readout is presented in Hz.

When the cursor is turned on, the amplitude at its location replaces the full scale annotation. Comparing Figure 13B and 12D shows this replacement. The units of the readout between full scale and cursor amplitude are consistent.

The "wake up" state does not determine the status of the CURSOR TYPE, FREQ READS, or CURSOR POSITION controls. Further details of cursor operation are discussed in Section 16 and in Appendices A and B.

SECTION 14

OUTPUT TYPE CONTROLS

14. OUTPUT TYPE

FIGURE 14

Figure 14 illustrates the OUTPUT TYPE controls. These controls specify the target for data output by the 660. Output targets include analog and digital devices.

Depressing the CRT ONLY pushbutton cancels all other forms of output from the instrument. CRT ONLY is the default status of the instrument. Unless another pushbutton in the output type group is depressed, output of the instrument is to the internal CRT only.

Depressing the AUX pushbutton causes digital or analog output to an auxiliary device. The nature of the

transmission is specified through the DEFINE AUX key in the calculation control group. The button is depressed to cause digital output to the optional Model 134A Digital Plotter. With alternative entries to the DEFINE AUX, depressing this pushbutton can cause analog output equivalent to the current cursor to appear on the analog plotter terminals (rear panel) or transmission through the RS232, IEEE488, or high speed parallel interfaces. Details of the define auxiliary entries are contained in Section 16 and Appendices A and C. The LED on the AUX pushbutton remains lit for the duration of auxiliary data transmission. The transmission may be terminated manually by pressing the AUX pushbutton a second time, causing its associated LED to go out.

The PLOT SET-UP pushbutton is used in conjunction with an optional analog X-Y plotter. The first time this pushbutton is depressed, an output voltage is applied to the horizontal and vertical plotter outputs corresponding to the lower lefthand corner of the display. The second time this pushbutton is depressed, the voltage at those terminals is changed to correspond with the upper righthand corner of the display. Repeatedly depressing this switch causes the voltages to alternate between the lower left and upper right

positions.

When a two trace display is drawn, depressing the PLOT SET-UP pushbutton steps the voltages on the horizontal and vertical plotter outputs through a sequence corresponding to:

1. Lower lefthand corner of upper trace
2. Upper righthand corner of upper trace
3. Lower lefthand corner of lower trace
4. Upper righthand corner of lower trace

To exit the PLOT SET-UP mode, any of the remaining OUTPUT TYPE pushbuttons may be depressed. Normally, either the PLOT or CRT ONLY pushbuttons are depressed.

Depressing the PLOT pushbutton causes analog outputs necessary to draw the data of the screen image on an analog plotter to appear on the rear panel vertical and horizontal plotter outputs. Analog plotter output consists of data traces only, no graticule or annotation information is output.

When a plot is initiated by depressing the PLOT pushbutton, the CRT image remains unchanged. A single intensified dot is seen to scan across the CRT image, indicating the position of the analog plotter's pen. At the conclusion of the plot, this dot remains at the extreme righthand side of the dis-

play and the voltage outputs on the rear vertical and horizontal axis terminals correspond to the last data point in the display.

Following a plot, depressing the CRT ONLY pushbutton clamps the plotter output levels to the zero volts.

In addition to horizontal and vertical axis plotter outputs, the 660 provides a pen lift output. This output is a switch closure used for lowering the pen on an analog plotter. When the PLOT button is depressed, the switch closure is made. Following a plot, when the CRT ONLY pushbutton is depressed the switch closure is opened.

In the "wake up" state the 660 is placed in the CRT ONLY mode. The define auxiliary is specified to support the 134A Digital Plotter via RS232.

SECTION 15

DISPLAY MEMORY

15. DISPLAY MEMORY Controls

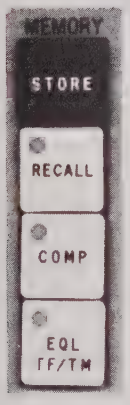


FIGURE 15A

The DISPLAY MEMORY controls are shown in Figure 15A. These controls permit storage, recall, comparison, and equalization of data. A single screen image of one or two traces may be stored in its entirety. A stored screen image may be recalled for display at a subsequent point in time. When data is recalled, the display reflects the control status of the instrument at the time the data was stored, even though control changes may have been made between the time the data was stored and recalled.

Facility is provided to compare the stored measurement with its currently measured equivalent. When two traces

have been stored from a single screen image, the comparison is limited to two display traces, one from the currently measured function and one from the stored function. Either of the two stored traces may be compared.

If the stored information is a transfer function, it may be used to equalize a currently measured transfer function, transmissibility function, or impulse response. Equalization is a means of correcting for phase and amplitude shifts introduced by test measurement components external to the 660. This function is particularly useful when analyzing data which has been tape recorded.

When the STORE pushbutton is depressed, the currently displayed screen image, one or two traces, is stored in a separate display storage memory. The entry/recall field indicates that storage has occurred by displaying the message DATA STORED as shown in Figure 15B.

Depressing the RECALL button causes the stored display to replace the currently measured display. Depressing this pushbutton a second time reverts the display to the currently measured function that was on display before the RECALL button was first depressed. When the RECALL button is depressed, the screen annotation reflects the status of the instrument at the time of storage. All parameters that may

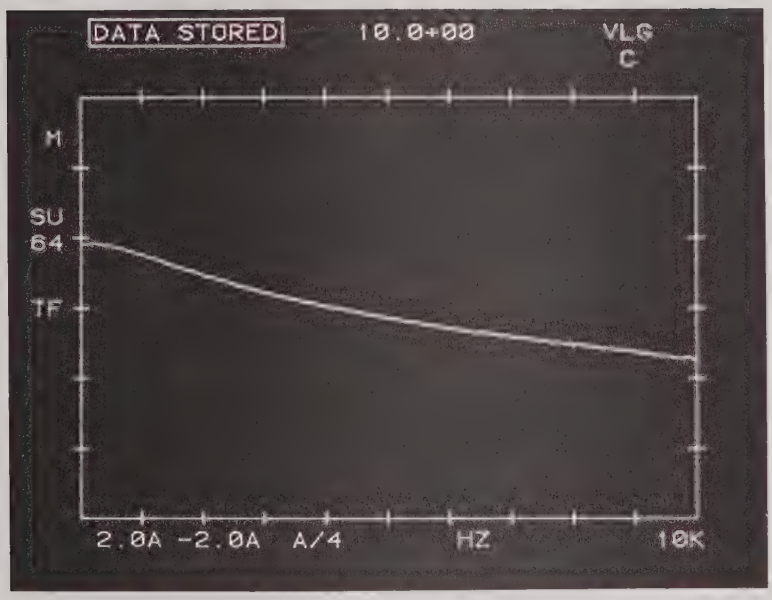


FIGURE 15B

be recalled through the entry/recall field reflect the status of the instrument at the time the data was stored. When the RECALL button is depressed the second time, the currently measured function returns to the CRT with all parameters, gains, etc. restored to the condition that they held prior to the recall of the stored information.

Depressing the compare (COMP) pushbutton causes the display of the stored function and its currently measured equivalent as shown in Figure 15C. In Figure 15C the magnitude of two transfer functions is compared. The upper trace is always the current measured function, while the lower trace is the stored function. Note that only two traces may be displayed at one time. Hence, with a transfer function or any complex function or pair of single channel functions stored, the comparison must be made one trace at a time.

Comparisons can only be made between like functions acquired under similar conditions. For example, a probability density function cannot be compared with a transfer function. For this reason, if the COMP pushbutton is depressed and the currently displayed function does not match the stored function, the entry/recall field will display the error message RECALL, MATCH. This instruction advises the user to recall the stored

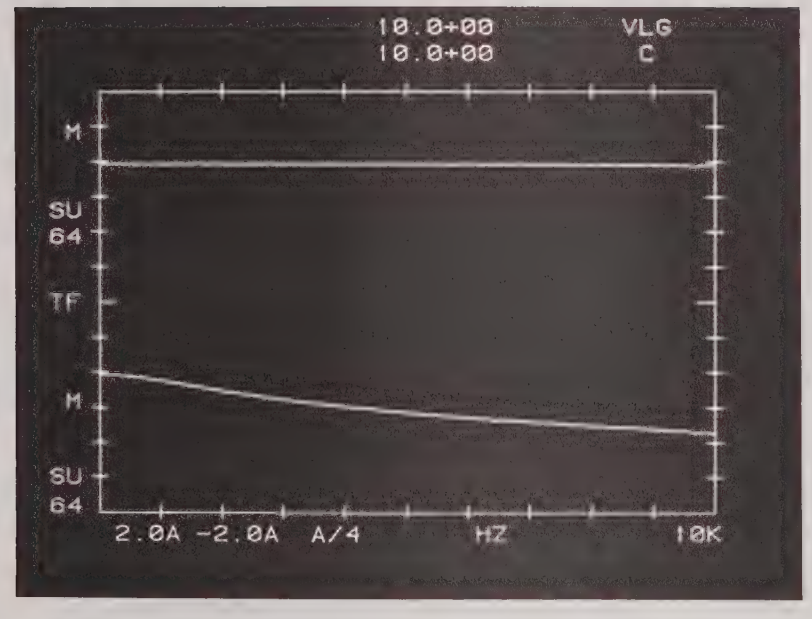


FIGURE 15C

information and then display its currently measured equivalent before depressing the COMP pushbutton.

Attempting comparisons with channel AB or format RI or format PHASE MAG or format Nyquist will produce the error message ONLY 2 TRACES. This message instructs the user to select the channel or format of the currently measured function to produce only a single trace before depressing the COMP button.

If a transfer function has been stored, the equalize (EQL TF/TM) pushbutton may be used. If the stored data is not a transfer function, the error message TF NOT STORED will be presented in the recall field. Equalization can be applied to the transfer function, transmissibility function, and impulse response. No other functions can be equalized.

Depressing the equalize pushbutton performs the complex division (equalization) of the currently measured transfer (or transmissibility) function by the previously stored transfer function. Complex division is equivalent to dividing the magnitude of the currently measured transfer (or transmissibility) function by the magnitude of the stored transfer function. Additionally, the phase spectrum of the stored transfer function is subtracted from the phase spectrum

of the currently measured transfer function. Figure 15D illustrates the equalized transfer function display of Figure 15C. Note that the function label has been modified by appending the prefix E for equalized to the function label TF. As shown in Table 9-1 modified labels of ETF, ETM, and EIR are possible.

When the currently selected measurement function is the impulse response, the equalization operation is first applied to the currently measured transfer function. Next, this equalized transfer function is inverse Fourier transformed to provide the equalized impulse response (EIR).

Attempting to depress the RECALL, COMP, or EQ TF/TM pushbuttons without previously storing a screen image will result in the entry/recall error message NO STORED DATA. This message indicates that the user must first store a screen image before attempting to use one of these functions.

In the "wake up" state no data is stored.

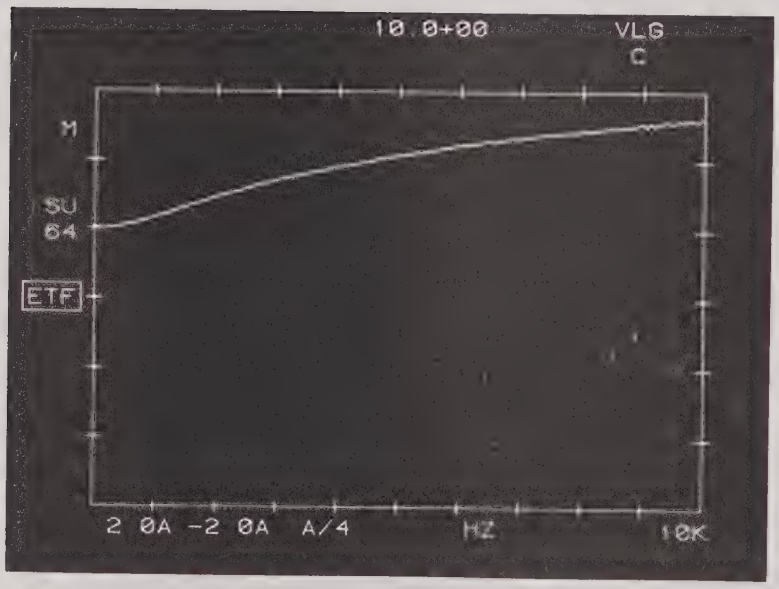


FIGURE 15D

CALCULATION CONTROL GROUP

16. CALCULATION CONTROL

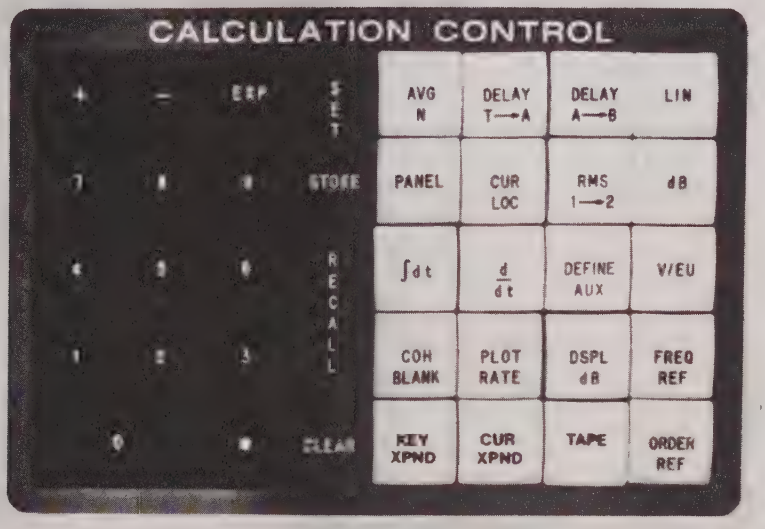


Figure 16A illustrates the calculation control group. This control section is used to enter numeric information to the processor, to recall operating statuses of the instrument, and to modify the behavioral performance of the 660. Transactions through the calculation control are displayed in the entry/recall field as they occur. Unless deliberately cleared, or otherwise used, the entry/recall field always displays the last transaction conducted with the calculation controls.

Three types of transactions may be conducted. These include:

1. Data may be entered using the SET/STORE key.
2. Information may be interrogated using the RECALL key.
3. An operation or characteristic initiated through a SET/STORE command may be returned to the "wake up" state condition by using the CLEAR key.

Data entries are always made with a left-to-right sequence. A number is typed, followed by touching the SET/STORE key, and normally followed by touching one of the twenty white buttons on the righthand side of the calculation control group. Data entries may be made in integer or floating point format. The EXP key is used to enter exponents. This key permits direct floating point entry. For example, the number 35,000 may be entered using the keystroke sequence 3, 5, EXPonent, 3 (35 times 10^3). All entered numbers are assumed to be positive unless the minus (-) key is depressed before depressing either the EXP or SET/STORE keys. Exponents, when used, are assumed positive unless the minus key is depressed after touching the EXP key and before depressing the SET/STORE key.

Leading zeros are ignored and the use of the decimal point is optional. There is no one correct way to enter a number. Any of several possible entries can be valid. For example, to enter the number -.018 all three of the following keystroke sequences are valid:

1. Minus, decimal point, 0, 1, 8
2. Minus, 1, 8, exponent, minus, 3
3. Minus, 1, decimal point, 8, exponent, minus, 2

With one exception, all data entries involve entering a number, followed by the SET/STORE, and followed by one of the twenty white pushbuttons. The exception operation is to explicitly set the display gain from the keyboard. This operation is conducted by entering the desired number of the display full scale, followed by touching the SET/STORE key. Next, before touching any of the white calculation control keys, touch the upper or lower gain cursor. The cursor may be touched in either the upward or downward direction. Immediately upon touching the gain cursor, its associated data trace will assume the full scale value entered through the keyboard. This facility permits entering numbers other than those in the nice full scale number list.

The use of the decimal point is optional in normal data entries. There is, however, one operation that is distinguished by requiring the decimal point to be entered. This operation is setting the cursor explicitly from the keyboard, rather than using the CURSOR POSITION cursor. To explicitly position the cursor, enter the desired cell number (1 to 400 for frequency displays, 1 to 1024 for time functions, 1 to 127 for amplitude functions), followed by the decimal point, SET/STORE, and the CUR LOC button. This entry will cause the immediate move of the cursor from its current position to the specified cell number.

Depressing the CLEAR key clears the entry/recall field. This key may be used to eliminate an erroneous numeric entry. Depressing the CLEAR pushbutton, followed by one of the twenty white calculation control keys will clear that function to its "wake up" state condition.

Depressing the RECALL key, followed by one of the twenty white keys will cause the previously entered value to be displayed in the entry/recall field. Thus, any parameter of significance can be brought to display prior to photographing or hardcopying the CRT image. Further, any numeric entry typed through the calculation control group without the following SET/STORE and function button sequence may be left on display

for hardcopy. This permits entering identification numbers on screen images.

The twenty white calculation control keys each have defined functions. These are listed, with a description of their use and their "wake up" state conditions, in Appendix A. The DEFINE AUX key is discussed in greater detail in Appendix C. Appendix B reviews entry/recall field messages that may be generated by improper use of the CALCULATION CONTROL . The following figures illustrate typical operations conducted through the calculation control group.

In Figure 16B the screen image of a captured transient time history is shown. The left most data point in this display is the first one which satisfied the transient conditions specified by the capture controls. In this instance, it is desired to view some information prior to trigger conditions.

In Figure 16C a similar transient has been captured after typing the sequence minus, .125, SET, DELAY T→A. This sequence established an anticipation (negative delay) between the trigger point and the first data point held. Notice that the capture point has been moved to 12.5% (.125) of the time window. Delays ranging from -1 to +63 time windows with resolution of 1/1024 windows may be entered through this key.

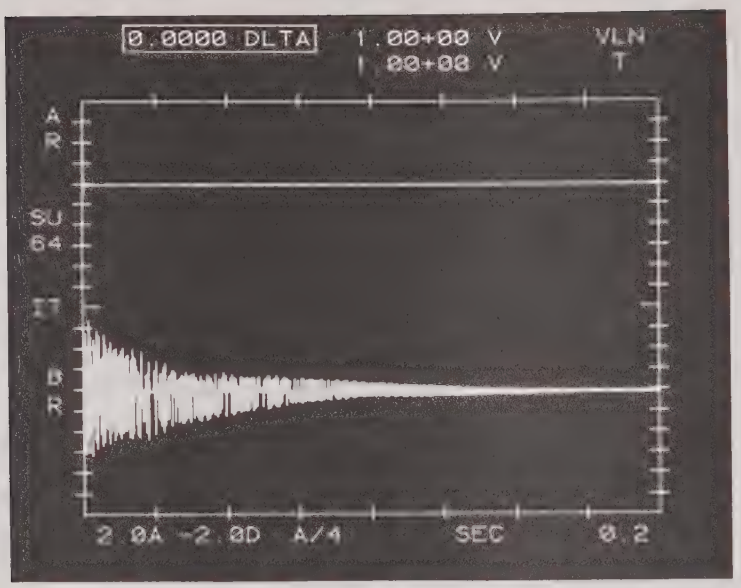


FIGURE 16B

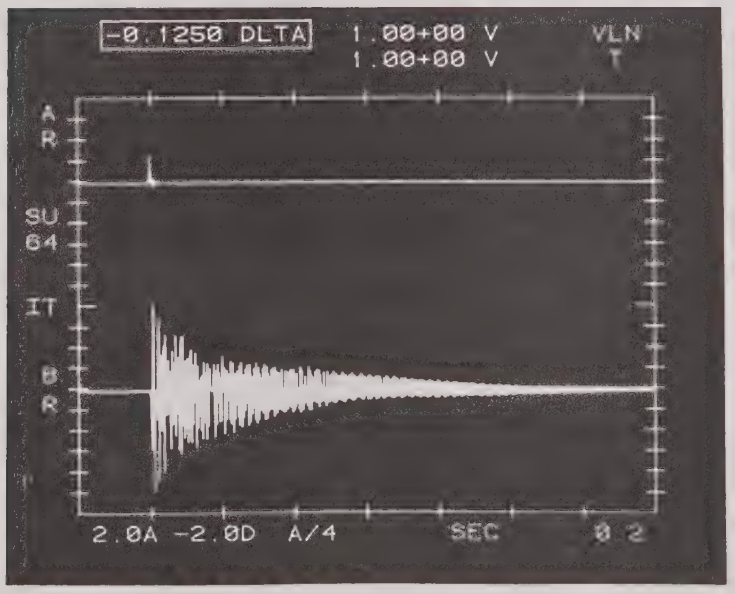


FIGURE 16C

Additionally, delays between channel A and B of 0 to 64 time windows may be entered using the DELAY A→B key.

Figure 16D illustrates an RMS spectrum. Figure 16E shows the same data after the keystroke sequence, 2 SET $\int dt$, has been made. This keystroke entry must be made while viewing a single channel frequency function. The result of this keystroke entry provides artificial integration for the viewed channel. One or two integrations may be specified. Artificial integration is performed by $j\omega$ division in the frequency domain. Thus, if the data in Figure 16D had represented an acceleration spectrum, then that in Figure 16E would have represented a displacement spectrum.

It is sometimes desirable to change the display range. Figure 16F presents the same data as that of Figure 16E, but the keystroke sequence, 80, SET, DSPL dB, has been entered. This entry changed the range of display from the "wake up" state of 60 dB to 80 dB. Selections from 10 to 80 dB, in 10 dB steps, may be made.

The coherence function is often used to qualify the believability of a transfer function (or other cross property function) as a function of frequency. This validation has been automated in the Model 660 by entering the keystrokes sequence, .9, SET, COH BLANK. A coherence blanking level of 90% has been specified. The transfer function displayed in

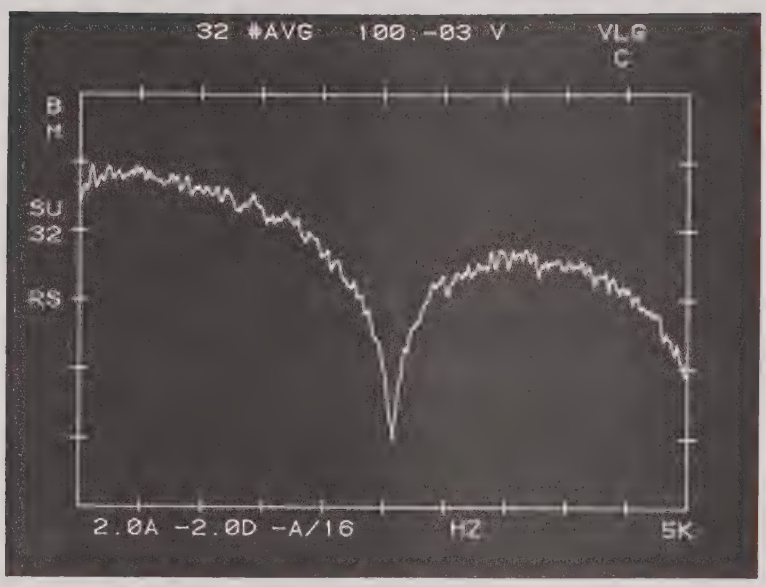


FIGURE 16D

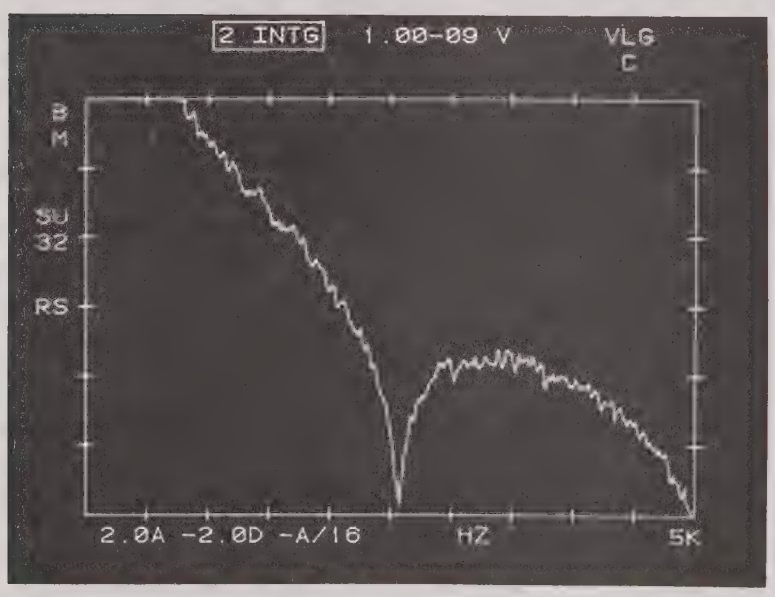


FIGURE 16E

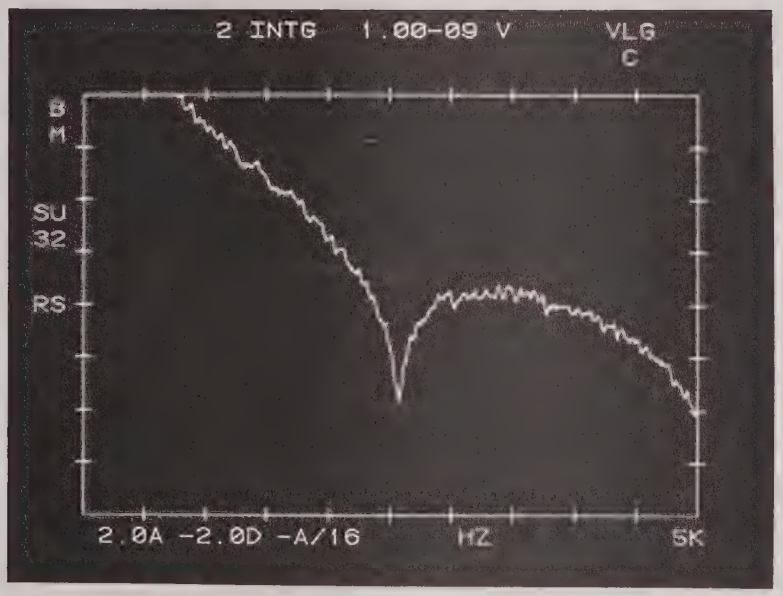


FIGURE 16F

Figure 16G is blanked at all frequencies where the associated coherence function is less than .9.

Other important calculation control operations include establishing volt/engineering unit scale constants. These may be entered by using the V/EU keys or the LIN and dB keys in conjunction with the cursor. The FREQ REF and ORDER REF keys are also used in conjunction with the cursor to modify horizontal readouts. The CUR LOC key may be used to store up to ten cursor locations for immediate recall through the keyboard. The entire operating status of the front panel may be stored for future use by using the PANEL key. This key permits storing up to ten entire front panel statuses so that they may be rapidly recalled with a single key sequence. Stored panel settings are retained even if the instrument is turned off.

Obviously, the calculation control group expands the versatility and flexibility of the 660. This group of controls is the key to tailoring the instrument's response to the user's needs. Please review the attached appendices so that maximum benefit may be obtained from the instrument.

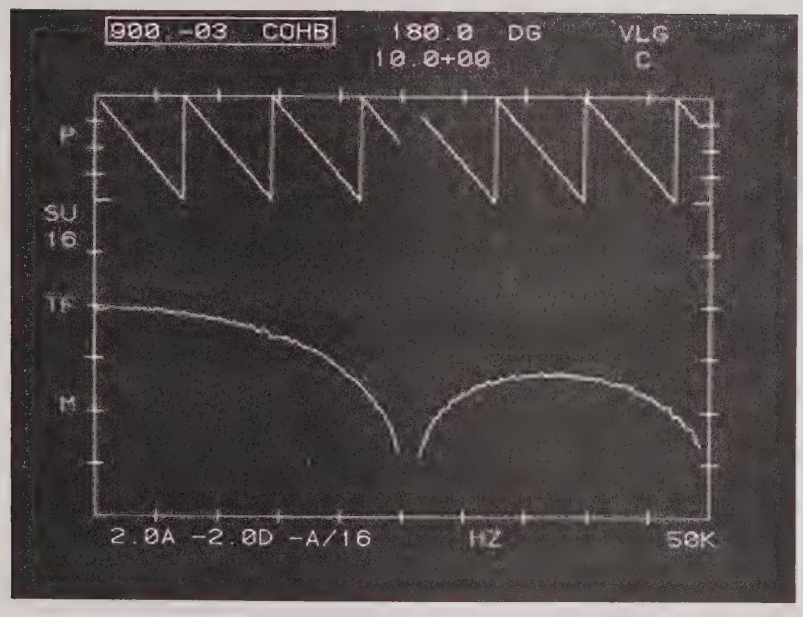


FIGURE 16G

SECTION 17

POWER ON CONDITIONAL STATES (WAKE-UP STATE)

17. Power On, System Reset, and Clear Panel State

In this state the 660 Spectrum Analyzer is set to the following, with NICOLET 660 appearing in the entry/recall field.

MODE - 1K CHA-B FFT

FUNCTION - INST SPEC

AVERAGE - SUM (Averager Off, # Avg = 0)

DISPLAY — CH - A/B

FORMAT - MAG

MEMORY - No lights on, no data stored

SCALES — AMPL - LOG

FREQ - LIN

OUTPUT TYPE - CRT ONLY

CURSOR READS - LIN, V, ABS, (PER Hz Off)

CAPTURE CONTROL - CONT, REL, (not in HOLD, not in ARM)

Cursor Location - Midscreen

In addition, the CALCULATION CONTROL settings are as follows:

AVG N - 16

DELAY T→A - 0

DELAY T→B - 0

V/EU - 1 V/EU A & B

Rel - 1

Int/Df - Off (0)

COH BLANK - Off (0.00)

PLOT RATE - 5 (Fastest)

DSPL dB - 60 dB

FREQ REF - 0 Hz

ORDER REF - 1 Hz/Order

The DEFINE AUX settings are:

0	1	Output to 134A Digital Plotter over RS232
1	0	50% Redundancy, Normal Trigger
2	0	Graticule, Annotation, and Data "On"
3	4	I/O Format: Binary, No Block Fill
4	0	Secondary Interface: 8 Bit Serial I/O
5	5	ASCII OUTPUT Buffer Terminator: (CR), (LF), (ETX)*
6	1	Handshake "On"

*Last Buffer Only

SECTION 18

INTERNAL SWITCHES

18. Internal Switches

Five screwdriver set, hexadecimal switches are located on the EXECUTIVE board inside the unit, see Figure 17. Reading from the front of the board these should be set to the following:

First (nearest the front)	E
2nd	0
3rd	0
4th	0
5th	3

NOTE

These switch settings are preset at shipment. They should not be changed without consulting Nicolet Scientific Corporation. Resetting these switches will not damage the instrument, but may result in operating difficulties.

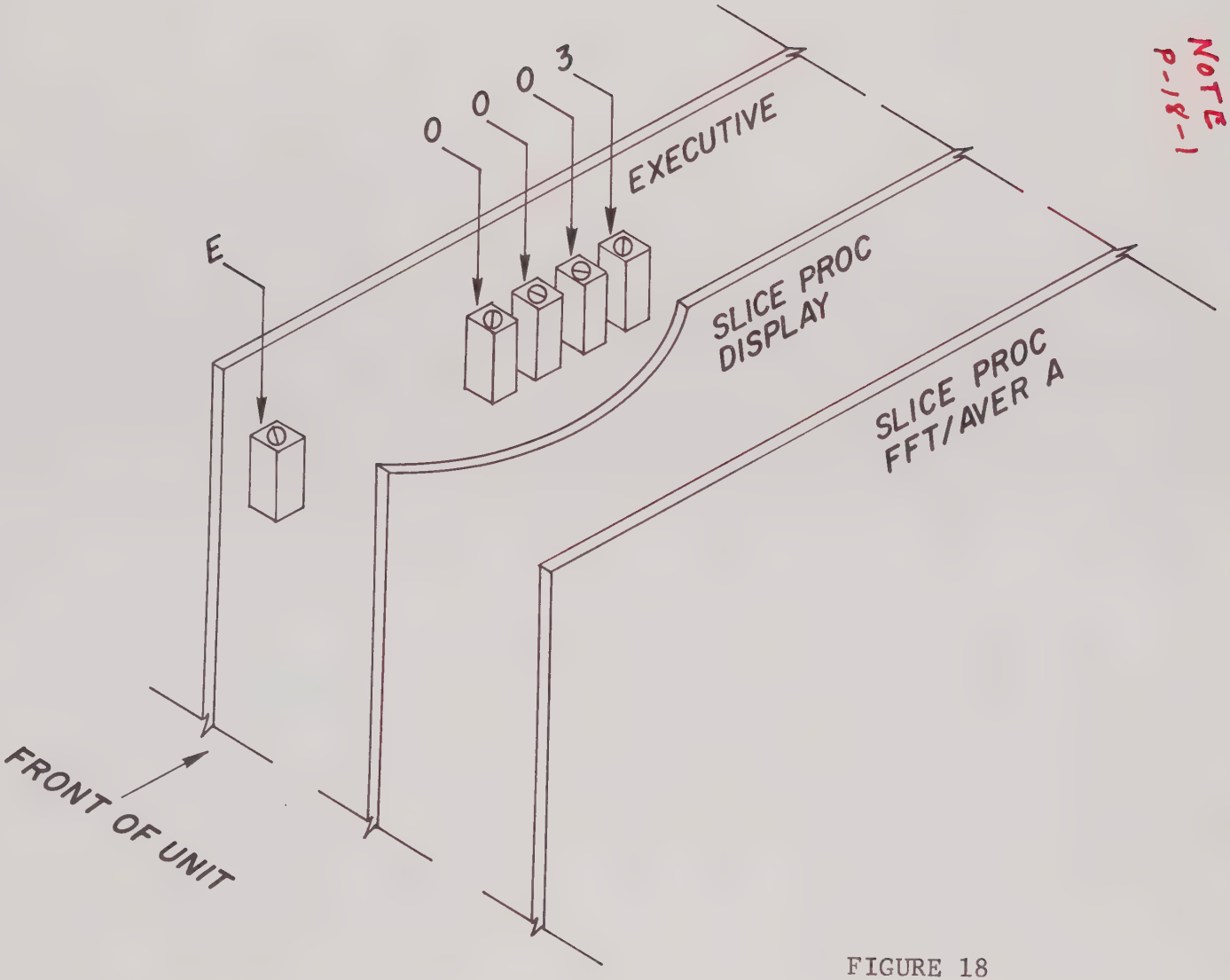

NOTE
P-18-1
0 0 0 3
EXECUTIVE
E
SLICE PROC
DISPLAY
SLICE PROC
FFT/AVER A
FRONT OF UNIT

FIGURE 18

APPENDIX A

MODEL 660 DUAL-CHANNEL FFT ANALYZER

CALCULATION CONTROL (KEYBOARD) OPERATION

The CALCULATION CONTROL keyboard is used to enter numeric data. As the data is entered, it appears in a field in the upper left of the CRT display.

In general, the numbers are entered in scientific notations, e.g. 1.23-3 = 1.23×10^{-3}. The sign may be entered before or after the number or exponent is entered, but the last sign entry takes precedence. At least 3 significant digits are stored, except 6 for time or frequency. Exponents must be no greater than 33. Leading zeros may be entered but are not necessary except after the decimal point.

Examples:	Key Entry	Number Set
	-1.23 EXP 3	-1.23×10^{3}
	01.23 -+ EXP 03-	$+1.23 \times 10^{-3}$
	0.123 - EXP + 3	-123×10^{0} (= -123)

The CLEAR key erases an entered number. CLEAR (function) returns function to initialized, start-up condition; RESET returns system to initialized conditions. NOTES:

(+) All numbers are assumed positive if no sign is entered.

* Indicates number is stored with panel setting with PANEL mode.

"Initialized" number is automatically set when the unit is turned on or when system RESET is pushed.

( ) After number indicates the range of possible numbers.

FUNCTION	SEQUENCE	DESCRIPTION
1a. Set N, the number of spectra in the average	*Number (1-9999) SET AVG N	Causes averaging to stop after N spectra in the SUM or DIFFERENCE averaging modes, or determines the averaging time constant (speed of response) in Exponential. (In Ex, the next lower power of "2" is automatically employed.) Averaging in PEAK and SWEEP modes continues until STOP is pushed, regardless of N. In Ex (only), can change N without stopping averaging. (Initialized to "16").
1b. Reads N	RECALL AVG N	Number of spectra averaged to the moment appears in the display.
2a. Set time from a trigger to the start of CH A time window	*Number (-1.000 to 63.000 for "1K") *Sign (-means anticipation) SET DELAY T→A	Delays (+) or anticipates (-) the capture of a transient from either an external trigger or a trigger level on the transient itself (see CAPTURE CONTROL) in terms of time windows. Resolution of the setting is 1/1024 of a time window or approximately .001 ("in 1K CH A-B mode"). The time window is determined by the setting of FREQUENCY RANGE. (For "2K CH A" mode, maximum is 31 time windows and resolution is 1/2048 time windows or approximately .0005).
2b. Read time from trigger	RECALL DELAY T→A	(Initialized to "000" time windows, i.e., no delay with the left edge of the time display the instant of triggering).

FUNCTION	SEQUENCE	DESCRIPTION
3a. Set time delay between CH A and CH B	*Number (0.000 to 64.000) DELAY A→B	When collecting and processing two channels of data, sets time delay of CH B data with respect to CH A data in terms of time windows in order to compensate for delays inherent in the experiment, e.g., transport delays. Resolution of setting is 1/1024 x time window or one sample. Time window is determined by setting of FREQUENCY RANGE.
3b. Read time delay from CH A to CH B	RECALL DELAY A→B	(Initialized to "0", or no delay).
4a. Set calibration at cursor in engineering units	Number EXP Sign Number (0 to 33) SET LIN	When displaying a single ch. INSTant SPECtrum or RMS SPECtrum, a known amplitude reading is assigned to a spectrum element identified with the cursor (cursor in LINear EU). Exponent seeks a multiple of "3" and the decimal point automatically readjusts for no more than 3 decimal places. Two different EU calibrations can be entered (ABS/REL). The RELative number can be entered to allow measurement of amplitude differences from one spectrum peak to another.

RECALL DEFAULTS TO ITEM 6

FUNCTION		SEQUENCE	DESCRIPTION
4b.	Measure in engineering units	(Reads at all times in the annotation in LIN-EU settings of CURSOR READS buttons)	SEE ITEM 6
5a.	Set calibration at cursor when reading amplitudes of frequency functions in dB	*Sign (+,-) *Number (0.0 to 299.9) SET dB	When displaying single channel INSTant SPECtra or RMS SPECtra, a known dB reading is assigned to a spectrum element identified with the cursor. Two different dB calibrations can be entered, one in ABSolute, one in RELative.
	RECALL DEFAULTS TO ITEM 6		
5b.	Read calibration in dB of frequency function	(Reads at all times in the annotation in dB-EU settings of CURSOR READS buttons)	SEE ITEM 6

FUNCTION	SEQUENCE	DESCRIPTION
6a. Set sensitivity of test system (including analyzer) in volts per engineering unit	*Number *EXP *Sign (+,-) *Number (0 to 30) SET V/EU	When displaying single channel INSTant SPECtra or RMS SPECtra, a known calibration equivalence is established between volts and engineering units. The exponent automatically seeks a multiple of "3" and the decimal point automatically readjusts for no more than 3 decimal places. (ABS/REL switch in CURSOR READS must be in ABS).
6b. Read sensitivity in V/EU	RECALL V/EU	Reads sensitivity of both input channels in any dual channel display or dual channel function. (Initialized to 1.0 V/EU).
7a. Retain settings of panel controls and keyboard	Number (1 to 9) SET PANEL	Stores all pushbutton settings and all keyboard settings indicated by "*". Also stores all other pushbutton settings (scales, modes, display, etc.). Does not store cursor location or AVERAGE START/STOP. Nine I.D. numbers are available. Total of 10 panel patterns can be stored (including "0", as defined below).
7b. Recall settings of panel controls and keyboard	Number (0 to 9) RECALL PANEL	Re-establishes panel control settings indicated by "*" for each of ten numbered patterns. Panel settings at moment of recall are stored automatically as "0".

FUNCTION	SEQUENCE	DESCRIPTION
8a. Retain current cursor location	Number (1 to 9) SET CUR LOC	Stores location of cursor in frequency displays when cursor is on.
8b. Recall past cursor location	Number (0 to 9) RECALL CUR LOC	When stored cursor locations are recalled, the present one is automatically stored as "0" and can be recalled as "0". (Initialized at mid-screen).
9a. Calculate RMS power between two frequencies	RECALL RMS 1→2	Power between cursor locations "1" and "2" inclusive is calculated and displayed in the annotation. If two spectra are displayed, both values are calculated. To obtain the total RMS in the entire spectrum cursor locations at the left and right edges of the gradicule are used. With cursor off, RMS power in entire spectrum display is calculated.
10a. Convert acceleration to velocity to displacement (typ) by synthetic input integration	*Number (0, 1, 2) SET $\int dt$	The effect of single (1) or double (2) integration of the data from CH A or B (set in single ch. display) is achieved by calculations in the frequency domain after processing; most commonly used to convert acceleration data to velocity to displacement. Applies to all subsequent displays or single or dual channel frequency domain functions. (If "0" is entered, previous integration is removed).

FUNCTION	SEQUENCE	DESCRIPTION
10b. Recall whether integration has been applied	RECALL $\int dt$	Number displayed is number of times (2 max.) the data in CH A or B (recalled in single ch. display) has been effectively integrated. (Initialized at "0" or no integration).
11a. Convert displacement to velocity to acceleration (typ) by synthetic differentiation	*Number (0, 1, 2) SET d/dt	The effect of single (1) or double (2) differentiation of the data from CH A or B (single ch. display) is achieved by calculations in the frequency domain after processing; most commonly used to convert displacement data to velocity to acceleration. Applies to all subsequent displays of single or dual channel frequency domain functions. (If "0" is entered, previous differentiation is removed).
11b. Recall whether differentiation has been applied	RECALL d/dt	Number displayed is number of times (2 max.) the data has been effectively differentiated. (Initialized at "0" or no differentiation).

FUNCTION	SEQUENCE	DESCRIPTION
12a. Select output for data transfer	Number (00-61) SET DEFINE AUX	(a) Selects output device, type of transfer, and format, (b) Selects format of output at cursor, (c) Turns off graticule and/or cursor, (d) Establishes primary and secondary interface, (e) Establishes type of device to be emulated See Separate Code Explanation, Appendix C.
12b. Read ouput selected	Number (0-6) RECALL DEFINE AUX	Places AUX code in recall field. See Appendix C.
13a. Set level of coherence, below which dual ch. freq. functions are blanked	*Sign (+ or -) *Number (.000 to .999) SET COH BLANK	Plus sign means blanking of dual ch. functions occurs only after averaging is stopped; minus sign means that blanking occurs during data collection while averaging. Zero means no blanking.
13b. Read the coherence blanking level	RECALL COH BLANK	(Initialized at "0" or no blanking).
14a. Set an X-Y plotter rate	*Number (1 to 5) SET PLOT RATE	1 = very slow, 2 = slow, 3 = medium, 4 = fast, 5 = very fast; all plot rates include automatic slow-down for large peaks to ensure faithful reproduction. (PLOT in OUTPUT TYPE starts X-Y plotting of any single display). (Initialized to 5).

FUNCTION		SEQUENCE	DESCRIPTION
14b.	Recall plotter speed	RECALL PLOT RATE	Places plot rate in recall field.
15a.	Set the range of the log display	*Number (10 to 80 in 10's) SET DISPL DB	Lower number provides greater vertical resolution but less viewing dynamic range (Gain level shifts the range of the log display "down" for greater sensitivity or "up" for less sensitivity in 10 dB steps); number of scale marks on the vertical axis indicates display range, one mark for each 10dB.
15b.	Recall log display setting	RECALL DISPL DB	(Initialized at "60").
16a.	Read frequency relative to a reference	Number (0 to 100,000.000 or to 1.000 x 10^5) SET FREQ REF	A reference frequency, identified with the cursor, is entered in Hz (directly or in scientific notation, e.g., "6.78 + 2 = 6.78 x 10^2 = 678), after which all subsequent frequency readings include the reference offset. For example, to read frequency modulation directly from sideband spacing, the cursor on the carrier might be assigned "0" Hz, after which the upper sideband might read "+25 Hz" and the lower "-25 Hz".

FUNCTION		SEQUENCE	DESCRIPTION
16b.	Recalls the frequency offset used in relative readings	RECALL FREQ REF	Reads the frequency in Hz (+ or -) of the display origin, based on SET-FREQ REF (16a above). (Initialized to "0", i.e. origin is 0 Hz). Clear only removes the use of frequency reference, does not return it to 0.
17a.	Provides a readings directly in "orders of rotation"	Number (0.0001 to 99.9999) SET ORDER REF	The order identified with the cursor is assigned a number and thereafter all readings (horizontal) will be referenced to it. For example, twice the frequency will be read as twice the set order, etc. Time domain displays will be read as "cycles" or "functions of a cycle".
17b.	Recalls maximum order displayed	RECALL ORDER REF	Reads Hz per order.

APPENDIX B

660 ENTRY FIELD MESSAGES AND ERROR INDICATIONS

(appear in upper left "entry field" in the annotated display)

MESSAGE	DESCRIPTION
NICOLET 660	Indicates that instrument has just been turned on.
BASEBAND DISP	Indicates that an expansion has just been cleared.
PANEL (N)	Indicates the current status of the instrument controls is panel pattern #N. (N is between 1 and 9).
CURSOR (N) XXXX	Indicates that #N cursor has been recalled. Address XXXX is the cursor location in "CELL" number, i.e., frequency element in a spectrum or sample increments in a time display.
FREQ ABSOLUTE	Indicates that the frequency reference has just been cleared.
DISP F. S. SET	Indicates that display Full Scale has just been set by explicit keyboard entry actuated by a momentary flip of either upper or lower gain switch (any 3 digit number with virtually unlimited exponents). As soon as lever "gain" switch is used again, only "preferred" numbers are employed.

ERRORS (reversed type)	OPERATOR HAS TRIED TO PERFORM THE FOLLOWING INCORRECT OPERATION
XXXX > YYYY	Enter a number not between the limits XXXX and YYYY.

ERRORS	OPERATOR HAS TRIED TO PERFORM THE FOLLOWING INCORRECT OPERATION
10 to 80 dB	Enter a display dB range outside of the allowable 10 to 80 dB limits.
10 dB STEPS	Set a display dB range to a non-integer multiple of 10.
NO EXPANDER	Set expansion when frequency expansion option (-2) is not in the instrument.
TF NOT STORED	Equalize without previously storing a transfer function.
NO STORED DATA	Compare or equalize without prior storage of data.
USE XXXX RANGE	Compare displayed function obtained on a range different from the stored function.
USE 1K FFT USE 2K FFT USE SIG ENH USE CORR USE PROB	Compare a function in an operating mode different from stored one.
VIEW FREQUENCY	Set a frequency or order reference while viewing a function other than frequency.
SET ORD > 0.0	Set an order reference equal or less than 0.00.
VIEW A OR B	Set an integration or differentiation operation or enter a volt-per-engineering unit scale constant while viewing a two channel display.
MUST VIEW A	View a channel B function when operating in a mode for which only "A" is available.

ERRORS	OPERATOR HAS TRIED TO PERFORM THE FOLLOWING INCORRECT OPERATION
MUST VIEW B	View a channel A function when operating in a mode for which only "B" is available.
TURN ON CURSOR	Perform an operation involving the cursor when it is not turned on.
CURSOR LIMITED	Cursor position requested does not fall on display.
ONLY 2 TRACES	Compare dual displays (that would have resulted in 4 data traces). Selected a "format" requiring 4 traces. Selected A & B channel requiring 4 traces.
RECALL, MATCH	Compare a displayed function which is not the same as the one stored.
OPERATION N/A	Operation not applicable to the system.
FORMAT N/A	Display format not available in current panel configuration.
FUNCTION N/A	Function not available in current panel configuration.

APPENDIX C

AUXILLIARY MODES (DEFINE AUX button)

FOR MODEL 660 DUAL CHANNEL FFT ANALYZER

Following is a list of currently assigned numbers for the auxilliary modes. The * indicates the initialized or wake-up condition, which is re-established every time RESET is pushed or power is turned on. Auxilliary modes are stored with panel patterns when #-SET-PANEL buttons are pushed. Numbers are exclusive only within a 10-digit range, i.e. only one AUX mode from 30 to 39 can be active at any one time. The procedure for setting the auxilliary mode is to push the #-SET-DEFINE AUX buttons in sequence for each function.

Code Number	Function
0	Ø analog output at cursor on J116
*1	Ø display output on RS232 (for digital plotter)
3	Ø digital output at cursor on RS232
4	Ø digital output at cursor on IEEE 488
*10	Normal triggering with 50% redundancy (when possible)
11	Normal triggering with 0% redundancy
12	Memory triggering with 50% redundancy (when possible)
13	Memory triggering with 0% redundancy

(Ø started by pushing the AUX button in Output Type Group)

Code Number	Function		
	Graticule	Annotation (Input field always active)	Data
*20	On	On	On
21	Off	On	On
22	On	Off	On
23	Off	Off	On
24	On	On	Off
25	Off	On	Off
26	On	Off	Off
27	Off	Off	Off

Data I/O Format

30	ASCII decimal, no block fill
31	ASCII decimal, block fill
32	ASCII hexadecimal, no block fill
33	ASCII hexadecimal, block fill
*34	Binary, no block fill
35	Binary, block fill
36	S0/S1 format (NSC test mode)
37	S0/S1/S9 format (NSC test mode)

Secondary I/O Specification

*40	Secondary I/O is 8 Bit Serial (RS232, IEE488) Character I/O
41	Secondary I/O is 16 Bit Parallel Binary Interface (J114)

ASCII Output Buffer Terminators

	ALL BUT LAST BUFFER	LAST BUFFER
50	None	None
51	None	ETX
52	CR	CR
53	CR	CR, ETX
54	CR, LF	CR, LF
*55	CR, LF	CR, LF, ETX

Code	Function
	ASCII Output Buffer Handshake
60	RS232 output handshake off
*61	RS232 output handshake on

Note: Buffer size can only be changed via RS232 or IEEE input. Default size is 72 characters. To change buffer size enter the following characters: Number of characters (12-2048) SWA.

A P P E N D I X D

(Follows)

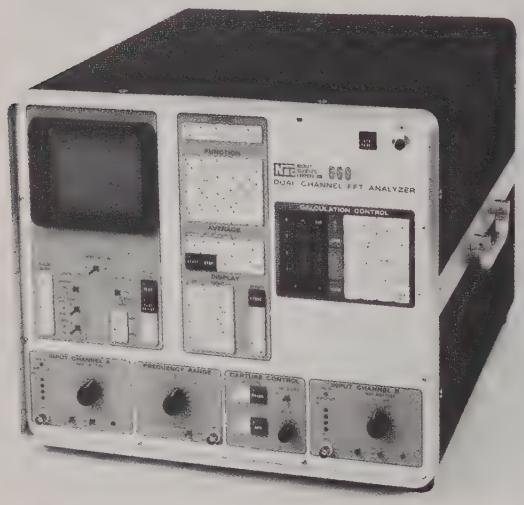
A P P E N D I X E

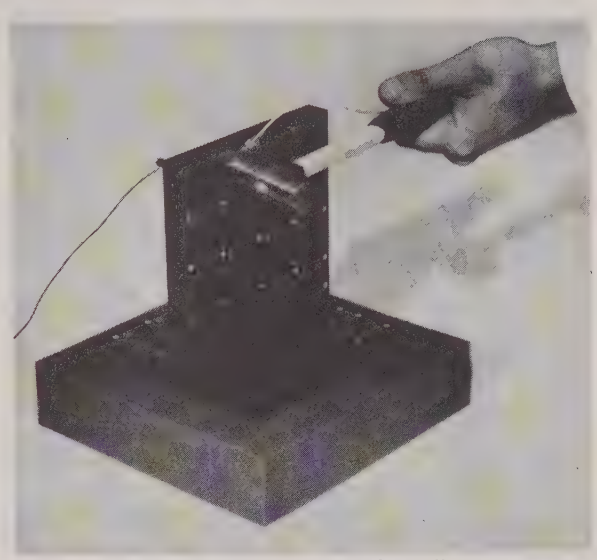
(Follows)

BUILT-IN 660 FEATURES AID STRUCTURAL ANALYSIS

(Impulse Testing of Structures for Modal Analysis)

The Nicolet Scientific Corporation Model 660 Dual Channel FFT Analyzer (below) contains many unique features designed specifically to simplify the analysis of structures. The following example using the structural characteristics of a simple aluminum T-plate highlights these features. In the example, an impulsive force input is used. (For additional technical background on such testing, consult Nicolet Scientific Corporation Application Notes 9 and 10.)





The tested T-plate is shown above. A small accelerometer is bonded to one corner of the plate, providing a motion output signal. A force is applied to the plate by striking it with a hammer, as shown. The face of the hammer is equipped with a piezo-electric force cell, providing an electrical signal proportional to the force imparted to the plate. The output of the force transducer is analyzed using channel A of the 660 while the accelerometer signal is analyzed using channel B.

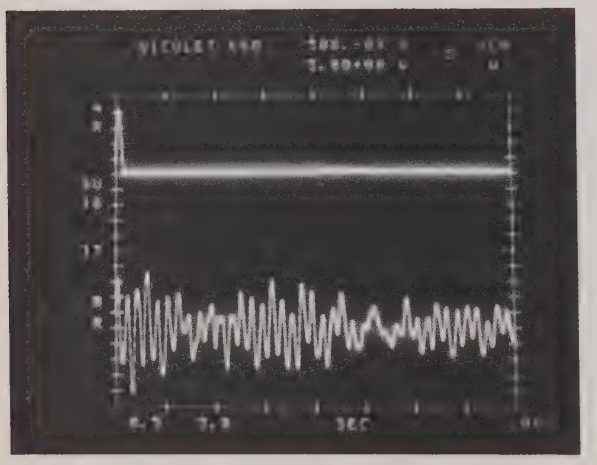


FIGURE 1: The time histories of a force pulse and resultant acceleration "ringing" are shown above. The analyzer is set to the 50 kHz range providing a time window of .008 seconds. Channel A sensitivity is 0.5 volts (0 to peak) while channel B is 5 volts. This "snapshot" is acquired by triggering the transient capture from the channel A data. The first point which satisfies the transient capture conditions is the left most point in the display. (The label "Nicolet 660" indicates that the processor is in its "wake-up" state, that is, power has just been applied to the instrument. In the "wake-up" state the analyzer automatically matches the display scale for amplitude and time to the selected input sensitivity of the instrument.)

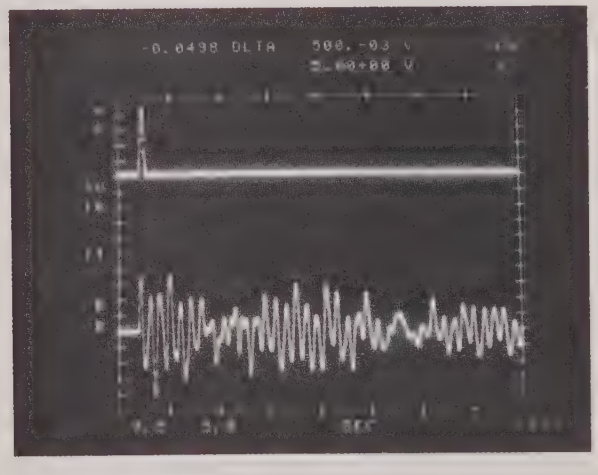


FIGURE 2: In order to observe the leading edge of the force transient, a trigger anticipation is set using the Calculation Controls. In this example, an anticipation of 51 samples (of the displayed 1024) is selected. This entry is denoted -0.0498 DLPA. The negative sign indicates that anticipation (negative delay) is required. Fifty-one samples corresponds to .0498 "time windows", or .0498 x 1024 samples. The mnemonic "DLTA" stands for "delay time" between trigger acquisition and channel A "hold".

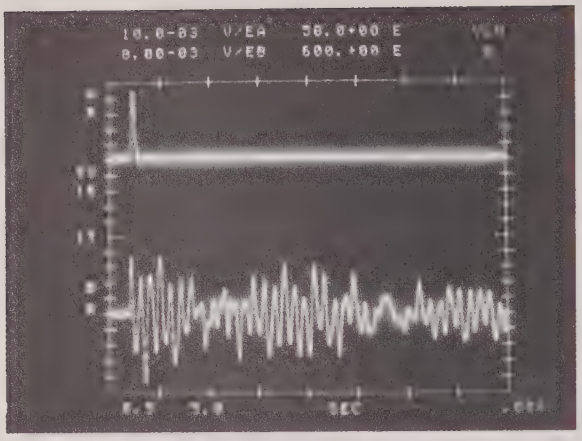


FIGURE 3: Of fundamental interest in this problem are "force" and "acceleration", not "voltage". The sensitivity of the transducers, 10 mv/lb for Ch A and 8 mv/g for Ch B, have been entered using the Calculation Control keyboard. These scale constants are indicated in the upper lefthand corner of the display. The displayed units of measure are also changed from V (for volts) to E (for engineering units). The full scale of the upper trace now corresponds to 50 pounds (0 to peak) while the full scale of the lower trace corresponds to 600 g's.

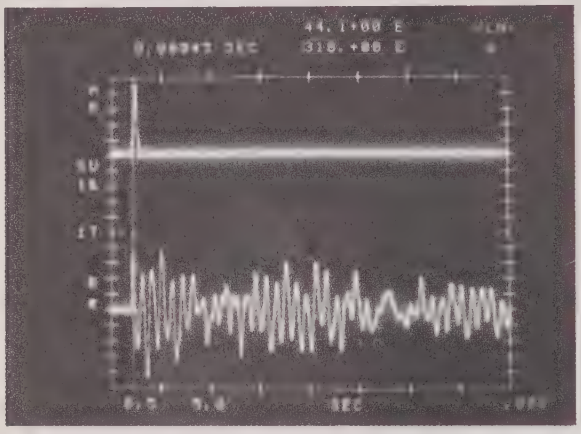


FIGURE 4: The time data on display is interrogated with a cursor and digital readout. The cursor is shown at a position corresponding to .00045 seconds (from the start of the window). The amplitude of the upper trace (force) at this time is 44.1 pounds. The amplitude of the acceleration signal is 318 g's.

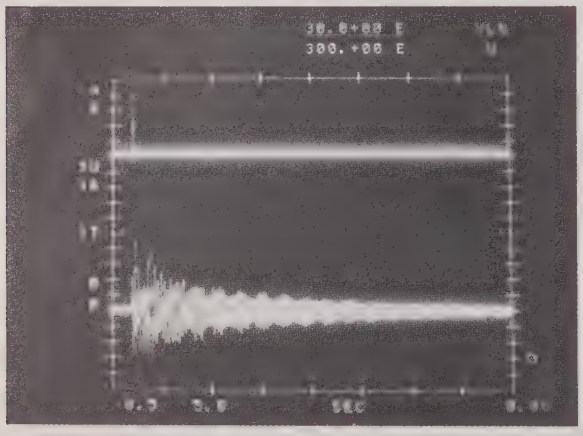


FIGURE 5: The frequency range of the analyzer is changed from 50 kHz to 5 kHz, providing a memory period of .08 seconds. This longer "snapshot" captures more of the "ringing" response of the acceleration signal.

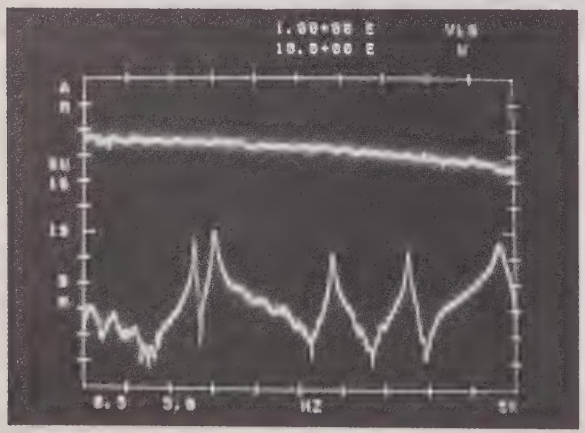


FIGURE 6: The top trace is the instant spectrum of the force signal (Ch A) and the lower one the acceleration signal (Ch B). The vertical axis is logarithmic. The full scale (top of graticule) for the upper trace corresponds to 1 pound (RMS). The full scale for the lower trace (center of box) corresponds to 10 g's RMS. There are five resonant "peaks" in the acceleration spectra.

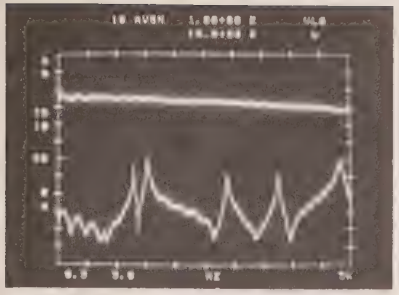


FIGURE 7: To enhance understanding of the structure, the results of ten hammer "raps" are averaged utilizing the transient averaging mode. Transient averaging is particularly convenient for this application because it automatically rejects any transients that overload the input to the instrument, allowing only valid transients within the range of the input analog-to-digital converter to be averaged. While the averaging is being conducted, the number of spectra actually averaged is displayed in the upper lefthand corner. The final averaged spectra are the true RMS spectra of the force and acceleration signals.

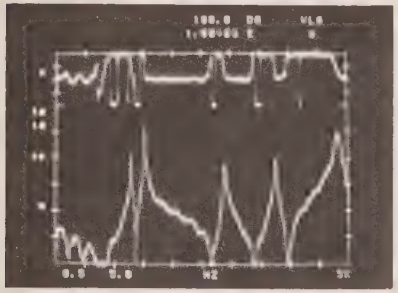


FIGURE 8: While averaging in dual channel mode, the instrument continually and simultaneously updates the power spectra for the force and acceleration signals as well as the cross-spectrum between them. Among many other functions, this permits the on-line, real-time computation of the transfer function, relating acceleration to force as a function of frequency. The upper trace represents the phase (in degrees) between the force and acceleration signals as a function of frequency. The lower trace is the ratio of acceleration to force as a function of frequency. The upper trace spans $\pm 180^{o}$; and the lower trace has a full scale of 1000 g/lb and a display range of 60 dB (1000:1).

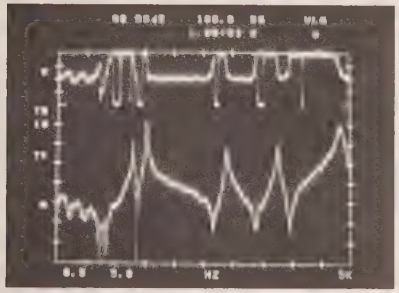


FIGURE 9: To enhance the transfer function interpretation, the range of display for the lower trace is increased to 80 dB (10,000:1). Display range may be selected from 10 to 80 dB in 10 dB steps.

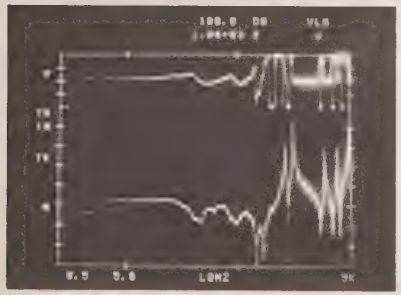


FIGURE 10: It is sometimes convenient to plot structural transfer functions in log/log form. Figure 10 is the same information as Figure 9 with the frequency axis changed from linear to logarithmic.

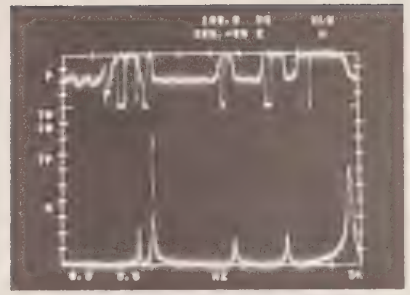


FIGURE 11: This figure shows exactly the same data as Figures 8 through 10 but with a linear vertical scale, identified by the code "VLN" in the upper righthand corner. Logarithmic displays bear the legend "VLG" in the same position. Note that a linear display makes the resonant "peaks" dominant. A logarithmic display shows the resonant "peaks" and the anti-resonant "valleys" with equal emphasis.

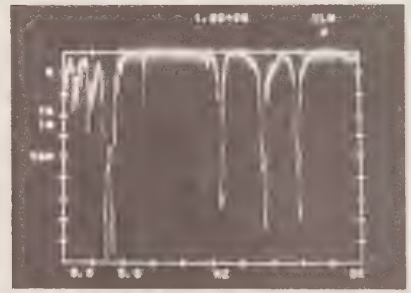


FIGURE 12: The confidence that can be placed in the value of a transfer function (at any frequency) is dependent upon the degree of linearity of the structure (at that frequency). The degree of linear causality is indicated by the coherence function, as shown. The coherence function is a spectrum whose amplitude is always between 0 and 1. Zero coherence indicates an entirely non-linear relationship between the force and motion of the structure, a value of 1 indicates a perfect linear relationship. In practice, information with coherence of .9 or greater is normally deemed acceptable. The coherence function may be thought of as a "guide" as to where (in frequency) the transfer function may be believed.

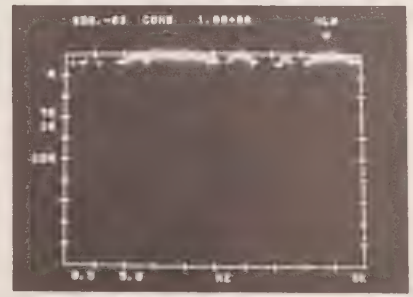


FIGURE 13: The 660 has a unique feature called "coherence blanking". In this figure the coherence function of Figure 12 is blanked for all values below .95. This is indicated by the notation "950.-03 COHB"

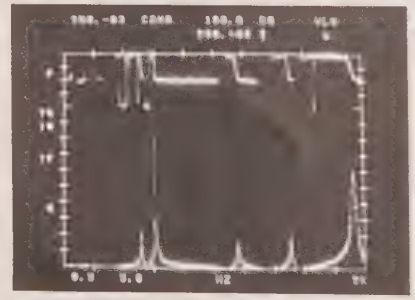


FIGURE 14: The coherence blanking level may be applied to all cross-property functions. Here it has been applied to the transfer function (as previously displayed in Figure 11). Note that both the amplitude and phase information of the transfer function are blanked so that only those frequencies where the coherence value is above .95 are shown. This useful feature permits validating the transfer function on a frequency-by-frequency basis without having to make tedious visual comparisons between the transfer function and coherence function.

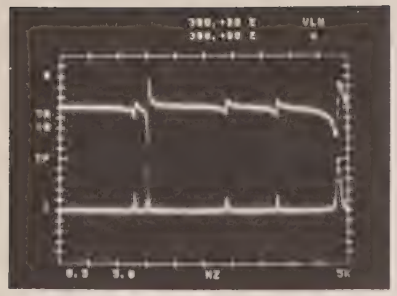


FIGURE 15: It is often useful to display a motion/force transfer function in terms of its real and imaginary components. (These are sometimes referred to as "co" or coincident and "quad" or quadrature components.) The real component of the transfer function is the product of the transfer function amplitude and the cosine of the phase angle between force and motion. The imaginary component is the product of the amplitude and the sine of the phase angle. This type of display is useful because the imaginary component exhibits an extremely sharp "spike" at exactly the undamped natural frequency of each mode. The real component exhibits a sharp minimum and maximum pair bounding each natural frequency. Identifying the frequency location of these maxima and minima permits extremely precise evaluation of the damping exhibited by the structure in each mode.

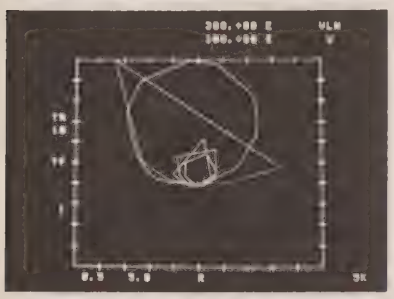


FIGURE 16: It may also prove useful to plot the real vs. the imaginary component to produce a Nyquist display. (By convention, the imaginary component is plotted on the vertical axis and the real component on the horizontal axis.) For proportional damping, the Nyquist display shows a circular "loop" for each natural frequency detected. The symmetry of these loops (with respect to a vertical axis) is often indicative of the type of damping the structure is exhibiting. As with most simple structures, the above figure indicates proportional damping.

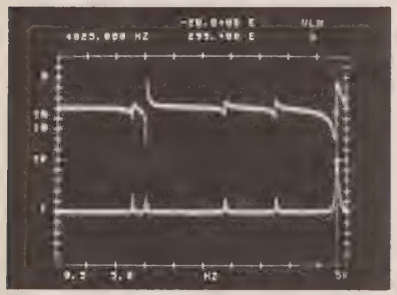


FIGURE 17: The cursor may be used to precisely measure amplitude and frequency. Here the cursor is located at 4825 Hz. The real component of the transfer function has an amplitude of -20.8 g/lb at this frequency, while the imaginary has an amplitude of 299 g/lb. For convenience, up to 10 cursor positions may be stored so that the cursor may be rapidly returned to pre-defined frequency positions. This facility is particularly useful when analyzing a structure's mode shapes, where the amplitudes of the imaginary components must be measured at each natural frequency from many transfer functions.

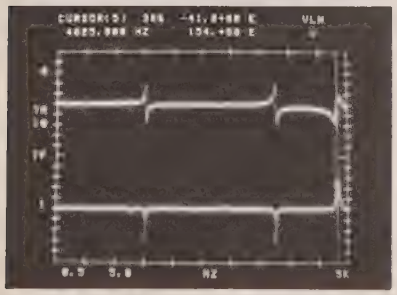


FIGURE 18: In all of the previously described transfer functions, the measurement was made by striking the T-plate at the same position where the accelerometer was mounted (i.e. in the upper lefthand corner). If the accelerometer position is left unchanged and the structure is "rapped" at different locations, the transfer function will change. Regardless of where the structure is impacted, "spikes" will always appear at the natural frequencies. The amplitude, however, of these "spikes" will change from position to position. These changes are indicative of the mode shape or vibration deformation pattern associated with each natural frequency. In Figure 18, the structure has been "rapped" at a different position from that of the accelerometer. A previously stored cursor position (the 5th) has been recalled using the keyboard. The annotation "cursor (5) 386" indicates that the 5th stored cursor has been recalled, and that it is at the 386th (of 400) frequency location. This location corresponds to 4825 Hz.

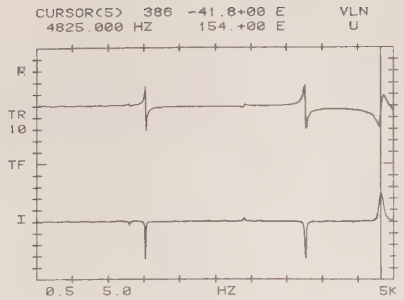


FIGURE 19: The screen images of the 660 provide valuable information for hardcopy storage. Figure 19 is such a plot, drawn with a Model 134A Digital Plotter. This instrument can reproduce all screen images of the Nicolet 660 in their entirety, including cursor location, annotation, graticule, and coherence blanking. The above figure is a plotted reproduction of Figure 18. (The preceding illustrations were recorded with the Model 61 Polaroid Recording Adapter.)

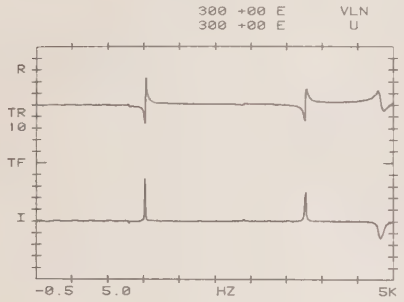


FIGURE 20: When making a large number of transfer function measurements from a structure it is often desirable to apply all the forces in the same direction. Unfortunately, this is not always physically possible. The 660 provides a simple feature to electronically "change the direction" of a force input. This is accomplished by electrically "inverting" the input to the analyzer. Figure 20 is shown with the force input (Ch A) inverted. The inversion is indicated by the change in sensitivity notation from 0.5 volts to -0.5 volts as shown in the lower lefthand corner of display. Note the change in the resultant transfer function between Figures 19 and 20.

In summary, features including transient capture positioning, engineering unit scaling, display annotation, cursor readout, display form flexibility, coherence blanking, cursor storage, input inversion, and digital plotting make the 660 ideal for structural analysis. This brief example merely "skims" the useful features of the 660 as a mechanical analysis instrument. It's compact, single unit construction permits it to be taken "on-site". Exclusive non-volatile panel setup storage (10 entire panel setting patterns may be stored and recalled even if the instrument has been unplugged and transported between storage and recall) simplify operation by less skilled personnel. Four types of digital interfaces (standard equipment) enhance the flexibility of the processor by allowing its ready interface to computers, calculators, and other digital peripherals. A broad range of special purpose options designed specifically for the analysis of mechanical problems further enhances the desirability of the 660.

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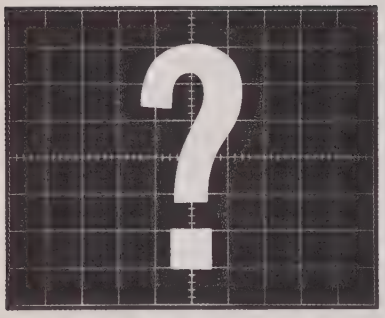
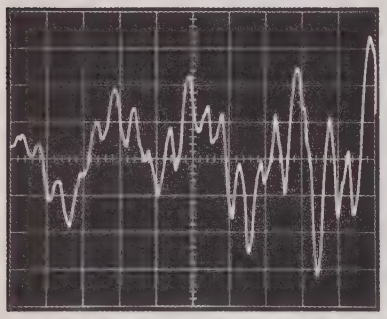
A P P E N D I X F

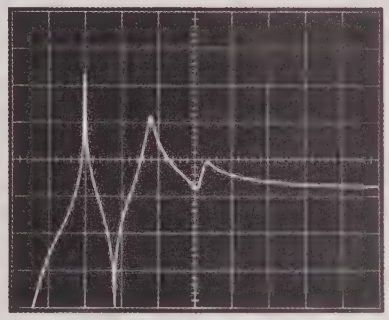
(Follows)

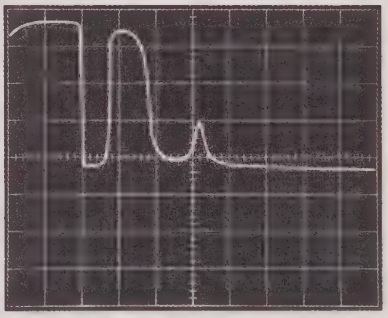
APPLICATION NOTE 9
JUNE 1975



UNDERSTANDING VIBRATION MEASUREMENTS







DYNAMIC BEHAVIOR OF STRUCTURES IS EXPLAINED BY TRANSFER FUNCTIONS AND MODAL ANALYSIS . . . KNOW HOW TO INTERPRET RESULTS.

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SUMMARY

The vibration of mechanical structures under environmental loading is a function of the nature of the loading *and the characteristics of the structure.* Characteristics of a broad range of structures can be well explained by *linear mathematics.* This fortunate situation can be capitalized upon by describing the structure's behavior with a series of *transfer functions.* Information extracted from such measurements facilitates improving the performance of a structure in its environment.

Structures vibrate harmonically; hence *frequency analysis* of motions and forces often provides more information than time histories do. The frequencies at which a structure tends to vibrate freely are determined by its distribution of *inertia* and *stiffness.* The degree to which these vibrations can be excited by external forces is determined by the structure's *damping* or energy dissipating characteristics.

A *transfer function describes a cause/effect relationship* between two measured signals. Experimentally exciting a structure with a measured force and simultaneously measuring its response motion permits a force/motion transfer function to be evaluated. This measurement can be used to determine the structure's resonant frequencies, damping characteristics, stiffness and inertia.

A *series of such measurements can be used to perform modal analysis,* defining the deformation pattern associated with each natural vibration. Understanding these modal deformations is essential if the structure is to be modified to change its vibratory characteristics. If the structure is to be attached to another structure, knowledge of the mode shapes is required for predicting the dynamic behavior of the composite structure.

This application note reviews the six classic mechanical transfer functions; their measurement, presentation and interpretation. Methods are presented for measuring resonant frequencies, mass, stiffness, damping and mode shapes. The modal analysis of a cantilever beam, using impulsive input, is discussed in detail.

INTRODUCTION

Mechanical structures transfer force and motion, which by nature are inseparable. The transfer of one variable always accompanies the transfer of the other and in product they constitute mechanical energy. Structural mass, stiffness and damping characteristics all impede the transfer of forced dynamic motion. These structural characteristics tend to delay or distort the force or displacement quantity being transferred. Understanding the nature of this transfer process is fundamental to understanding the behavior of a structure under its environmental loading situation.

The manner by which force and motion are related in a mechanical structure can be characterized by a *frequency domain transfer function.* A transfer function defines:

1. The ratio of sinusoidal force/motion as a function of frequency
2. The degree of phase lag or delay introduced by the process as a function of frequency

THE SIX TRANSFER FUNCTIONS

Six transfer functions have become classics in the study of mechanical structures. These are:

1. Compliance (displacement/force)
2. Apparent stiffness (force/displacement)
3. Mobility (velocity/force)
4. Impedance (force/velocity)
5. Inertance (acceleration/force)
6. Apparent mass (force/acceleration)

The *compliance, mobility and inertance* functions are all ratios of response motion to input force excitation. These functions all identify resonant frequencies by exhibiting "spikes" or maximum values at the occurrence of a resonance.

The *apparent stiffness, impedance and apparent mass* are the reciprocal relationships of the compliance, mobility and inertance, respectively. These functions denote a resonant frequency as a "valley" or minimum value. Structural antiresonances (nodes) appear in these functions as maximum values or "spikes."

All six of the descriptive transfer functions are interrelated by simple algebraic operations. *Having knowledge of one of the six frequency response forms is equivalent to having knowledge of all six.* The nature of available instrumentation, local conventions and history, and problem specifics normally dictate the selection of one of the six functions for measurement.

DRIVING POINT AND TRANSFER MEASUREMENTS

Structural transfer functions may be thought of broadly as being of two types:

1. *Driving point structural responses* define the relationship between force and motion at a single point in the structure as a function of frequency.
2. *Spacial transfer response functions* define the interrelationships between two points in the structure as a function of frequency.

In general, when completely defining a laboratory problem, both types of measurements are required. The driving point functions may be recognized as a specialized case of the spacial transfer relationship.

Physically realizable structures possess a symmetry known as structural reciprocity. The motion at a position in the structure due to a force at a second position is equal to the motion at the second position due to a force at the first position. This symmetrical characteristic means that the interrelationship N points on a structure may be characterized by $N (N + 1)/2$ transfer relationships. (In the absence of structural reciprocity N^2 relationships would be required to define the structure.)

PRESENTATION OF MEASUREMENTS

Certain forms of presentation of the transfer function relationships have become classically accepted. Any of the six forms of transfer relationship may be expressed as a *magnitude* (ratio of output to input amplitude) and *phase* (phase lag of output relative to input) versus frequency. A magnitude and phase description is called a polar display.

The same information can also be expressed in terms of two output/input ratios, one for the components in phase coincidence and the second for the components in phase quadrature. This *real* (coincident) and *imaginary* (quadrature) description is frequently referred to as a rectangular or Cartesian description.

Data from frequency response measurements are normally presented as plotted spectra with either linear or logarithmic presentation of the amplitude and frequency axes. Logarithmic amplitude displays tend to compress the data so that detailed information is available at both large and small output/input ratios. Linear displays of amplitude tend to accentuate resonant points (antinodes) in compliance, mobility and inertance presentations and antiresonances (nodes) in apparent stiffness, impedance and apparent mass presentation. Presentation of the real and imaginary components is especially useful in the precise location of resonant points. When dealing with systems exhibiting nonlinear damping characteristics, cross plotting the imaginary versus real components of the transfer relationship may be used to characterize the nature of the damping relationship. This cross plot is frequently called a *Nyquist diagram.*

COMPARISON OF PLOT FORMS

Figures 1 through 9 are typical driving point measurements made from a simple structure to illustrate these points. Figure 1 is a linear display of the amplitude of dynamic inertance, while Figure 2 displays the associated phase relationship. (Note: the inertance function has been normalized by multiplying it by the effective mass at the driving point.)

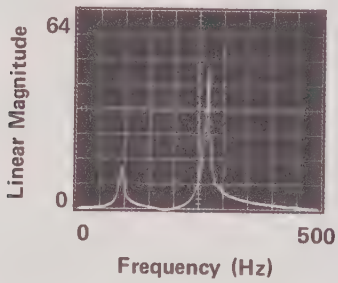


Figure 1: Inertance X Effective Static Mass

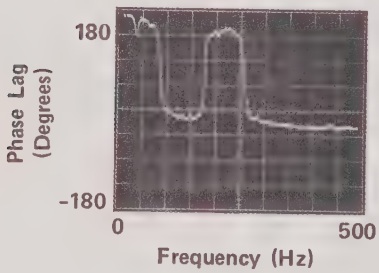


Figure 2: Inertance X Effective Static Mass

Figure 3 illustrates the same information with logarithmic amplitude (calibrated in dB). Note how the logarithmic display expands detail around the antiresonance. The same phase information is present as illustrated in Figure 4.

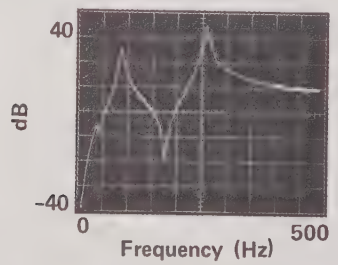


Figure 3: Inertance X Effective Static Mass

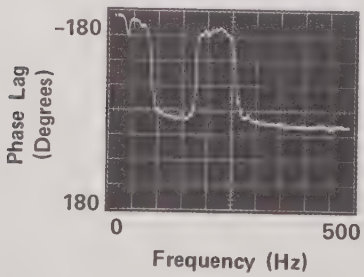


Figure 4: Inertance X Effective Static Mass

Figures 5 and 6 illustrate the real (coincident) and imaginary (quadrature) components of the inertance (with linear amplitude). Note that at the resonant frequencies the real component has a zero value while the imaginary components achieve a local minimum. *The real component achieves a local maximum below each resonance and a local minimum above it.*

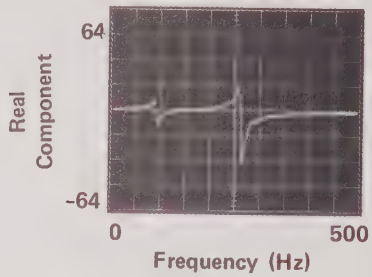


Figure 5: Inertance X Effective Static Mass

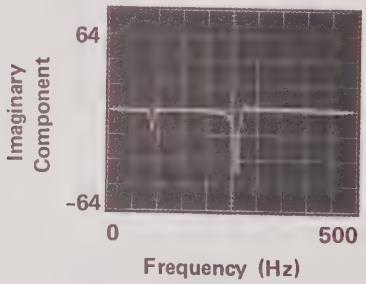


Figure 6: Inertance X Effective Static Mass

Figure 7 is a cross plot of the real and imaginary components. In this display one loop is found for each resonance encountered. *For ideally linear behavior, each loop is a circular form* tangent to the origin and centered about a point on the imaginary axis. The distortion noted in Figure 7 indicates the presence of non-linear restoring and damping forces in the simple structure because of the presence of hysteretic damping. Resonant frequencies can be detected in this cross plot by finding that portion in the display that exhibits maximum change in arc length with change in frequency.

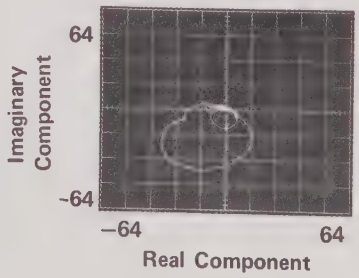


Figure 7: Inertance X Effective Static Mass

Figures 8 and 9 illustrate the logarithmic amplitude and phase lag of the dynamic mass function associated with the same structure. This function is the reciprocal of the previously discussed inertance function. Note that when presented logarithmically the dynamic mass and inertance functions appear as mirror images of one another reflected about a 0-dB line. (When displayed linearly, these functions

do not exhibit this simple "mirroring" relationship.) Note that the phase angle associated with the dynamic mass and inertance measurements are always mirror images of one another about the 0 degree line. While the inertance function identifies resonant frequencies as maximum values, the dynamic mass identifies them as minimum values. The maximum values in the dynamic mass function are associated with antiresonances or nodes. The nodes appear as minimum values in the inertance function.

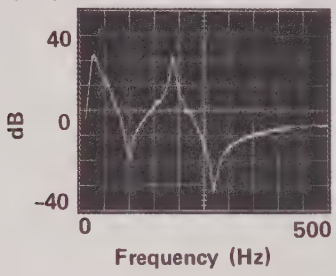


Figure 8: Dynamic Mass/Effective Static Mass

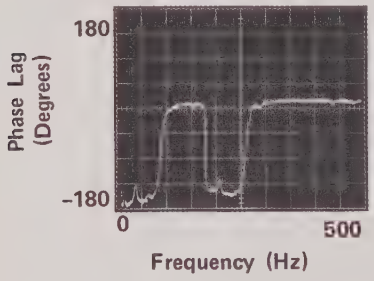


Figure 9: Dynamic Mass/Effective Static Mass

COMPARISON OF COMPLIANCE, MOBILITY AND INERTANCE

Figures 10 through 18 compare driving point compliance, mobility and inertance measurements made at a single point in a structure. Three resonances are evident as shown by the peaks in Figure 10, 12 and 14. Note that the associated phase data (Figures 11, 13 and 15) exhibits a near 180° shift each time a resonance is crossed.

All these functions show peaks at the resonant frequencies of the structure. In general, higher frequency modes tend to exhibit less displacement than lower frequency resonances. This is evident in Figure 10. Measuring velocity (rather than displacement) biases the measurement in proportion to frequency. Hence, the mobility magnitude (Figure 12) for higher modes is increased. The inertance magnitude (Figure 14) is biased in proportion to frequency squared as acceleration is measured.

The phase information of all three functions is identical. Measuring velocity or acceleration rather than displacement introduces a phase offset of 90° or 180°, respectively. This is illustrated by Figures 11, 13 and 15.

Figures 16, 17 and 18 illustrate Nyquist plots of the same data. Amplitude scaling emphasized the upper two resonances in these figures. Note that the 90° phase shifts between these functions appear as 90° rotations of the figures. Note further that the high frequency data increases in radius (magnitude) for mobility and inertance measurements.

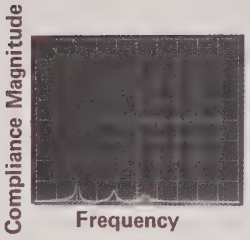


Figure 10

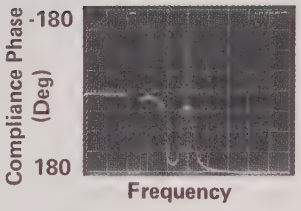


Figure 11

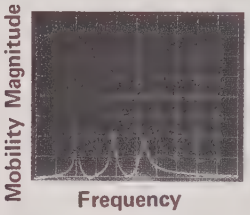


Figure 12

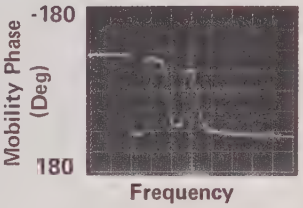


Figure 13

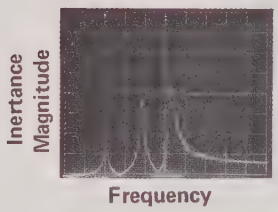


Figure 14

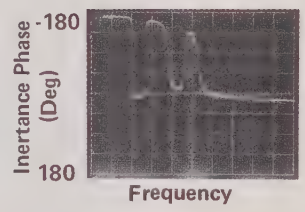


Figure 15

Figure 16

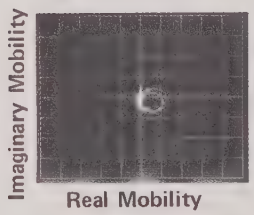


Figure 17

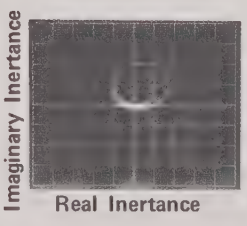


Figure 18

UNDERSTANDING RESONANT FREQUENCIES

The simple spring/mass/damper model shown in Figure 19 provides considerable insight into the behavior of more complex structures. *If there is no damping in the system (C = 0) the structure vibrates freely at the undamped natural resonant frequency,* f_n, where:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}} \qquad \text{(Hz)}$$

With damping in the system, free vibration occurs at a lower frequency, f_d, *and decays in an exponential fashion.* The damped resonant frequency, f_d, is defined:

$$f_d = f_n\sqrt{1 - \zeta^2}$$

where the critical damping factor, ζ, is defined:

$$\zeta = \frac{C}{2\sqrt{KM}}$$

Figure 20 illustrates the change in f_d as ζ increases.

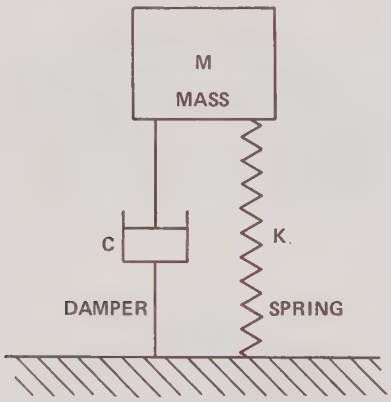


Figure 19: Single Degree of Freedom Spring/Mass/Damper System

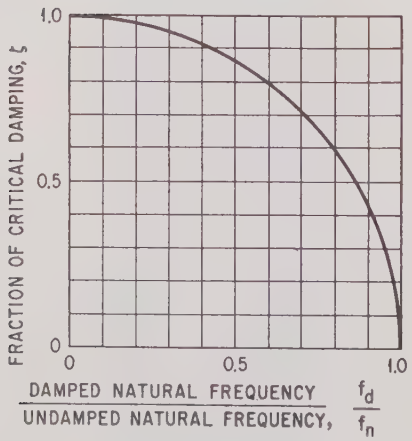


Figure 20: Effect of Damping on Natural Resonant Frequency (Single Degree of Freedom System)

When the damped system is excited by an external forcing function the *maximum response does not occur at the damped resonant frequency.* In fact, the resonant frequencies indicated by the compliance, mobility and inertance functions will be:

Peak compliance	$f_n\sqrt{1 - 2\zeta^2}$
Peak mobility	f_n
Peak inertance	$\frac{f_n}{\sqrt{1 - 2\zeta^2}}$

This difference in peak function response between the compliance, mobility and inertance is shown in Figure 21a. The mobility maximizes at a forcing frequency equal to f_n; the compliance maximizes at a lower frequency; the inertance at a higher frequency. *As the damping factor decreases these three frequencies converge to* f_n.

Most structural resonance problems involve modes that are lightly damped ($\zeta \leqslant .1$). For this reason *the peak value in a compliance, mobility or inertance function closely approximate* f_n.

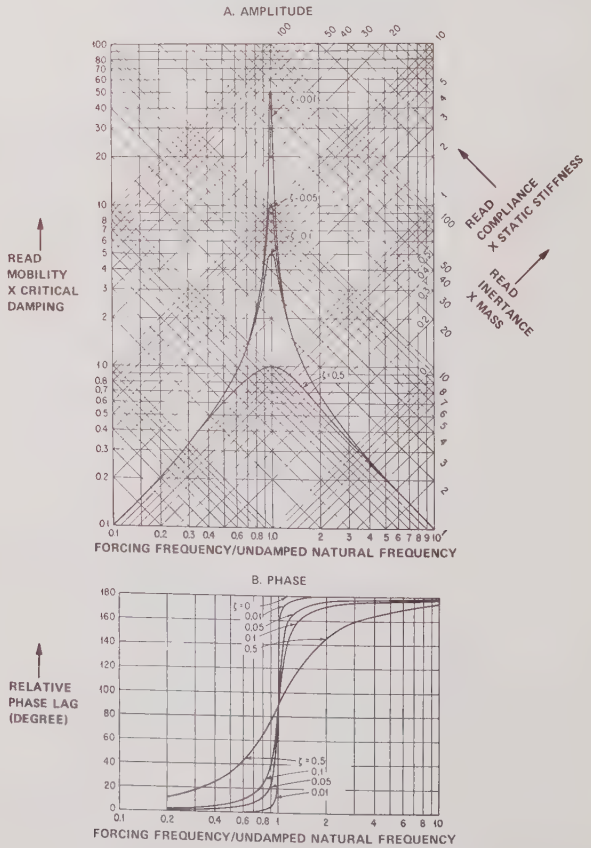


Figure 21: Comparison of Normalized Compliance, Mobility and Inertance Functions for Single Degree of Freedom System

All three functions exhibit a 90° relative phase shift at exactly f_n as shown in Figure 21b. At f_n the mobility has only *real* amplitude while the compliance and inertance have only *imaginary* amplitude. *This property may be used to detect resonant frequencies with greater precision.*

MEASURING MASS, STIFFNESS AND DAMPING

Plotting any of the frequency response functions with logarithmic amplitude versus logarithmic frequency provides a means of directly estimating mass, stiffness and damping parameters in systems characterized by far coupled modes. (When two or more resonant frequencies are closely spaced in frequency, these approximations are not precise enough to give useful information.) Such plots are frequently called *Bode plots.*

Basically, each resonance is characterized by three frequency zones of activity. At frequencies below the resonance, the activity of the structure is controlled by stiffness terms, so that the predominant reaction to an input excitation force is caused by the restoring forces (stiffness) which are proportional to the displacement and in phase with it. At the resonant frequency all restoring forces come from the damping terms, which are proportional to the velocity of the driving point. At frequencies above the resonance, the restoring force is predominated by the inertial (mass) terms, which are propotional to the acceleration at the driving point.

For sinusoidal motion the velocity is proportional to the product of displacement and frequency, while acceleration is proportional to the product of displacement and frequency squared. Thus, the observed restoring forces at the driving point (the apparent stiffness) are constant (with respect to frequency) below the resonance, increase in proportion to frequency through the resonance, and increase in proportion to frequency squared after the resonance. When presented in log/log format these three relationships produce asymptotic slopes of 0, +1, and +2. Thus, the asymptotic slope values in a log/log presentation of any of the six frequency response functions can be used to estimate the values of stiffness, damping and mass.

This is accomplished by measuring the response amplitude at the intersection of an asymptote with .159 Hz (1 radian/sec). Table 1 summarizes the interpretation of asymptotes for each of the six frequency response functions.

Figures 22, 23 and 24 illustrate log/log plots of compliance, mobility, and inertance measured on a simple structure with three resonances. Note that the stiffness asymptote of the first mode is 0 slope in the compliance function, +1 in the mobility and +2 in the inertance. Because the modes are closely spaced in frequency, the remaining stiffness and mass asymptotes are not easily read. The damping asymptotes are not evident as all three modes are lightly damped. In fact, *damping asymptotes are rarely a precise indication of damping in typical simple structures.*

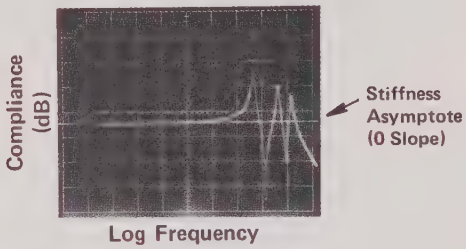


Figure 22

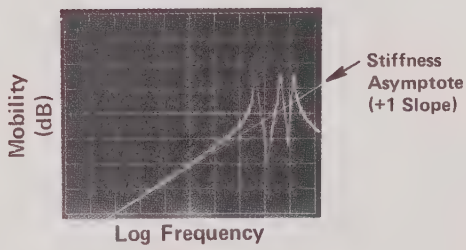


Figure 23

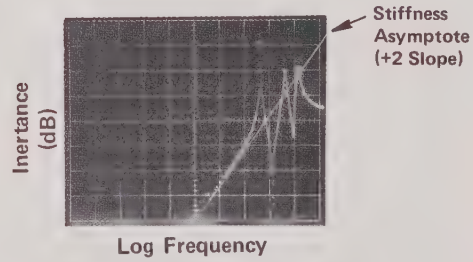


Figure 24

TABLE 1

	Asymptotic Slope				
Frequency Response Function	-2	-1	0	1	2
Compliance	$(Mass)^{-1}$	$(Damping)^{-1}$	$(Stiffness)^{-1}$		
Mobility		$(Mass)^{-1}$	$(Damping)^{-1}$	$(Stiffness)^{-1}$	
Inertance			$(Mass)^{-1}$	$(Damping)^{-1}$	$(Stiffness)^{-1}$
Apparent Stiffness			Stiffness	Damping	Mass
Impedance		Stiffness	Damping	Mass	
Apparent Mass	Stiffness	Damping	Mass		

The use of asymptotic approximations is widespread in mechanical industries. These approximations are often aided by plotting results on multigrided mobility paper as illustrated in Figure 25. This log/log graph form provides reference +1, 0 and -1 slopes to aid in the identification of mass, damping and stiffness terms.

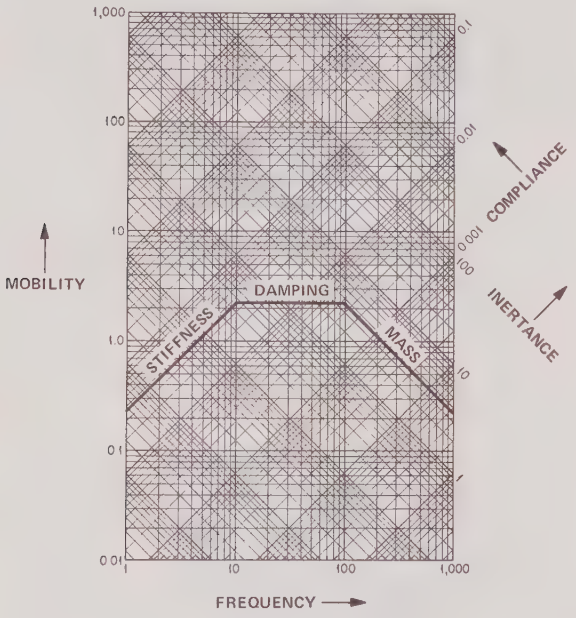


Figure 25: Directly Readable Mobility Plot Form Showing Asymptotes for Stiffness, Damping and Mass Terms

MEASURING DAMPING WITH GREATER PRECISION

In most structures the damping terms are not dominant; hence, their measurement is most difficult. The damping properties of a resonance are described by an amplication factor, Q. The amplification factor is related to the viscous damping factor, ζ , by the relationship:

$$Q = \frac{1}{2\zeta}$$

where

ζ = Damping ratio = C/C_c

C = Viscous Damping Coefficient (Force/Velocity)

and

C_c = Critical Damping Coefficient = $4\pi M_e f_n$

where

M_e = Effective Mass of Resonance

f_n = Natural Frequency of Resonance

If the structure exhibits hysteretic damping, it is damped by a force proportional to displacement and in phase opposition to velocity. This frequency independent damping is characterized by a nondimensional hysteretic damping factor, g. g is related to the amplification factor, Q.

$$Q = \frac{1}{g}$$

where

$$g = \frac{jF_D}{K_e}$$

and

$j = \sqrt{-1}$

F_D = Damping force acting at frequency, f_n

K_e = Effective stiffness of resonance

Q may be measured from frequency response functions by several methods. Three of the most popular methods are:

Method 1:

$$Q = \frac{|X/F|\,(f_n)}{|X/F|\,(0)}$$

where

$|X/F|\,(f_n)$ = Compliance magnitude at the natural frequency, f_n

$|X/F|\,(0)$ = Static compliance

Method 2:

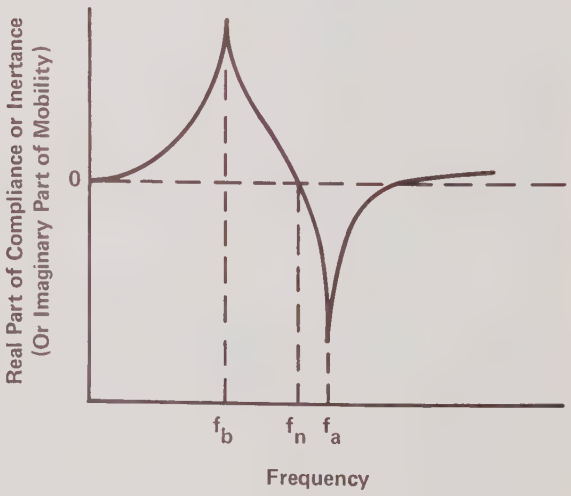
$$Q = f_n / \Delta f$$

where

Δf = Frequency interval between points that are 3 dB below the mobility magnitude at frequency, f_n

Method 3:

$$Q = \frac{(f_a/f_b)^2 + 1}{(f_a/f_b)^2 - 1}$$



where

f_a = *Frequency above resonance,* where the *real* part of compliance or inertance (or imaginary part of mobility) reaches a peak.

and

f_b = *Frequency below resonance,* where the *real* part of compliance or inertance (or imaginary part of mobility) reaches a peak *of opposite sign.*

Method 3 is the most precise in practical implementation, particularly when dealing with high Q modes. It has the distinct advantage of being directly applicable to either compliance, mobility or inertance functions.

MODAL ANALYSIS

The frequency response functions may also be used to characterize the shape of deformation of a structure associated with each natural frequency or resonance. *Each resonance is characterized by a unique distribution of deformation across the structure. This deformation is frequently referred to as a mode shape or normal mode.* (In a more mathematical sense the term eigenvector is synonomous with a mode shape.)

The resonant frequencies of a structure are independent of the spacial location from which they are measured. This is not true of the antiresonant frequencies. Figures 26, 27 and 28 illustrate driving point inertance functions measured at three different locations in the same structure. Note that the same resonant peaks are present in all three functions, while the antiresonant frequencies change with position.

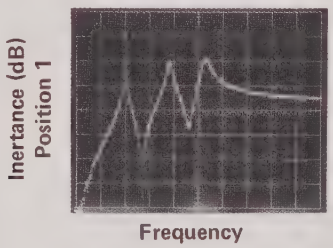


Figure 26

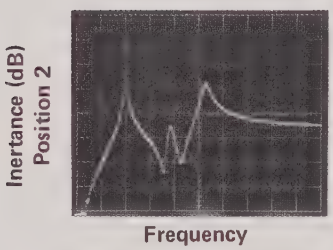


Figure 27

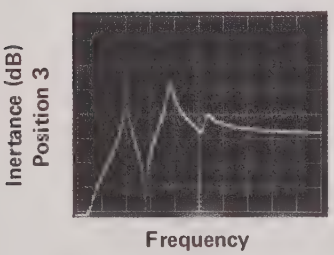


Figure 28

Each mode shape is a global property of the structure (as are the resonant frequencies) and affects motion over the entire structure. It is often desirable to determine the shape of deformation at a particular resonance to better understand how that resonant frequency might be shifted to another frequency location. *It is necessary to find those areas of the structure that undergo maximum motion in a particular mode, as these are the areas upon which corrective stiffness or mass terms will have maximum effect* (and, therefore, maximum opportunity to correct the problem).

Mode shapes are also of fundamental value in the analytical modeling of structures. Mathematical techniques exist which allow the prediction of dynamic behavior of complicated composite structures based on knowledge of the dynamic behavior of *individual* components. Substructures may be tested separately and mathematically "connected" together to predict the dynamic behavior of the composite structure.

Mode shapes can be estimated directly from frequency response functions. These estimated mode shapes give first order understanding of the nature of the deformation of a structure and are *frequently sufficiently precise* to allow the placement of corrective stiffness, damping or mass components. This is accomplished by *exciting the structure at a single point and measuring its response at several points.* Alternatively, the structure may be sequentially excited at a series of locations and the response measured at a single point. These two testing techniques are equivalent because of structural reciprocity.

Each excitation/response pair is characterized by a frequency response function. A modal measurement may be made most conveniently using compliance, mobility or inertance measurements. (The reciprocal nature of the apparent stiffness, impedance and apparent mass measurements tends to reduce the precision of the measure and adds an additional computational step.) In most structures, higher frequency modes tend to exhibit less response than the lower frequency modes. For this reason, the *inertance measurement is frequently the most desirable frequency response function* as it introduces a frequency squared bias favoring the higher frequency modes. This is illustrated by Figures 10, 12, and 14.

The mode shape is estimated by measuring quadrature (imaginary) response values from all of the measured inertance (or compliance) functions at each resonant frequency. At each resonant frequency, the quadrature components of the response functions are distributed in proportion to the displacement at each of the measured locations throughout the structure. That is, they are distributed in an approximation of the mode shape associated with that resonance.

A complete description of a mode contains four pieces of information:

1. f_n = Natural frequency on n^{th} mode.
2. Q_n = Amplification factor of n^{th} mode.
3. W_n = "Weight" of the n^{th} mode, an indication of how much structural mass is in motion in the n^{th} mode.
4. $\Psi m, n$ = Motion at the m^{th} location in the structure in the n^{th} mode.

Within a given frequency range, N resonances may exist. Hence, n ranges from 1 to N. If M locations with the structure are measured, then m ranges from 1 to M. *Each of the N mode shapes may be normalized so that the maximum motion (say at the p^{th} location) is equal to one unit* (inch, mm, etc). Other normalization schemes are possible, but this one simplifies plotting and interpreting results. The choice of mode shape normalization affects the modal "weight," W_n. The modal "weight" is defined by the summation:

$$W_n = \sum_{m=1}^{M} W_m (\Psi m, n)^2$$

where

W_m = Physical weight at m^{th} location

"HAMMERING OUT" A MODAL ANALYSIS

Although the frequency response functions may be construed as definitions of the sinusoidal input/output relationship of a structure, they can, in fact, be measured using non-sinusoidal excitation forms.* Three different techniques are broadly used. These test techniques utilize swept sine, random or impulsive excitation. Each has certain inherent advantages and disadvantages. There is no "right" technique for all problems. Fortunately, modern measurement instrumentation (such as the Nicolet Scientific OF 400A) are fully capable of performing analysis with any of the three types of excitation.

Impulsive testing is by far the most rapid technique that can be applied to the study of a complex structure. Its fundamental advantage is that no shaker system needs to be mounted to the test object. *Impulsive testing utilizes short duration transient force inputs (with corresponding broadband spectra) to excite all frequencies in the structure simultaneously.*

Impulsive testing is normally conducted using some form of an instrumented hammer (such as that shown in Figure 29). A force transducer is mounted to the head of the hammer, measuring the force input to the structure.

Figure 29: Instrumented Hammer Used to Impulse Structures

An accelerometer is used to measure the response of the structure. By far the most popular technique for impulsive excitation is to mount an accelerometer at a fixed location and to excite the structure at a multiplicity of locations using a hammer with a force transducer on its face to impulse the system at many locations. The data is analyzed with a dual-channel FFT analyzer. Each time the structure is struck with the hammer, both the impulse force and the response acceleration are captured by the FFT analyzer using the impulse force as a transient capture trigger condition. These two transient time histories are Fourier transformed to yield the input and output spectra. The resultant ratio of the output and input spectra is the desired frequency response function.

Usually, the results of several transient excitations are averaged. In this situation, the most desirable procedure is to compute the cross spectrum between the input and response and the power spectrum of the input. *The resultant averaged transfer characteristic is derived by dividing the cross spectrum by the input power spectrum* *(see page 15).*

The impulse hammer (excitation system) with sensitivity S_a(mv/lb) is attached to channel A of the FFT analyzer. The accelerometer (response transducer) of sensitivity S_b(mv/g) is attached to channel B of the analyzer.

Based upon a preliminary examination of the structure, the input attenuator of channel A is set to a full scale value of F_a and the input attenuator of channel B is set to a level of F_b. The levels of F_a and F_b can only be obtained by trial measurements on the structure.

The structure to be examined is marked at M spacial locations. The response transducer (accelerometer) is fastened to the structure at one location. (If at all possible, the response transducer should be located in a local "hot-spot," so that it will be capable of responding to all modes.)

Starting from the response transducer location, the structure is "rapped" a number of times at each spacial location. Each spacial location is "rapped" until the coherence function (a function which indicates the validity of the transfer function) approaches unity across the selected frequency analysis range. (Note, that the first "rap" will always produce a coherence function equal to unity.) It is essential that each location be "rapped" more than once, so that the coherence function may be used as a valid indicator of the quality of acquired data. The first "rapping" point is of particular importance.

At the first "rapping" point both stimulus and response readings are made at the same spacial location. The input/output relationships associated with this spacial location are used as a guide to interpreting all subsequent measurements. Specifically, the resonant frequencies of N modes are detected by identifying the N peaks in the *imaginary* part of the measured inertance function (acceleration/force transfer function). Additionally, the Q's of the N modes are measured using method three (described on page 8). Thus, the first measured transfer function identifies the parameter f_n and Q_n over the range of $1 \leqslant n \leqslant N$.

The imaginary values from the driving point transfer function at each of the N resonances should be measured and retained to normalize subsequent data. That is, the N quadrature points, $IMAG_{bb}$ (f_n) must be retained.

*See Nicolet Scientific Corporation's Application Note 10, titled "Shake, Rattle, or Rap?"

Driving point location, a, is subsequently moved through the sequence of predefined spacial locations. At each of the M locations the imaginary part of the transfer function is measured at the N preselected resonant frequencies. That is, quadrature values $IMAG_{a,b}(F_n)$ are measured and retained for all values of a between 1 and M and all values of n between 1 and N.

If all measurements can be acquired without changing the input attenuators of channel A or channel B, then a single scale factor is sufficient to relate physical and electrical units. Specifically, the constant, K, may be retained where:

$$K = -S_b F_a / S_a F_b$$

The coefficient, K, relates the voltage gain measured by the analyzer to physical measurements in the structure. Specifically, if the imaginary part of a transfer function, $IMAG_{ab}(F_n)$, is read at the n^{th} resonant frequency, it is realted to the physical parameters of the system by the equation:

$$IMAG_{ab}(f_n) = \frac{KQ_n \Psi_{a,n} \Psi_{b,n}}{W_n}$$

At any frequency, f_n, the stored quadrature components, $IMAG_{a,b}(f_n)$, will have a local minimum occurring at some spacial location a = p. The components $IMAG_{p,b}(f_n)$ are of particular importance. Specifically, these are used to compute the "weight" of each mode and to normalize the retained quadrature components from each mode to arrive at the spacial vectors $\Psi_{a,n}$. That is:

$$W_n = \frac{KQ_n \, IMAG_{bb}(f_n)}{IMAG^2_{p,b}(f_n)}$$

$$\Psi_{a,n} = \frac{IMAG_{a,b}(f_n)}{IMAG_{p,b}(f_n)}$$

These shape functions are approximations, because the measured shape (in response to a single point excitation) is actually a linear summation of all of the mode shapes of the system. However, if the resonances are broadly spaced in frequency and high "Q" in nature (Q = 5 or greater), then the interaction between the modes is minimal and the distortion of any given mode shape by contribution from other modes is not significant.

If the modes are closely spaced or of low "Q" then the interaction becomes significant and the estimated mode shapes are not precise. However, *this lack of precision usually does not affect their ability to direct where the structure should be modified to improve its performance.*

If increased precision of mode shapes is required, mathematical curve fitting techniques can be used to separate the interaction of the modes. These techniques require precise knowledge of the resonant frequencies and damping factors of each mode. Unfortunately, the precision with which these parameters can be measured deteriorates as modes become closely spaced and damping becomes more significant. Thus, *in many situations, the additional effort of mathematical curve fitting fails to significantly improve the precision of mode shape estimates.* Higher precision can be achieved by more complex mathematical techniques (i.e. regression analysis) requiring large scale computer systems for their efficient implementation.

A PRACTICAL EXAMPLE OF MODAL ANALYSIS

Figure 30 illustrates the geometry of a steel cantilever analyzed by a Nicolet Scientific OF-400A Fast Fourier Transform Processor using impulsive techniques. An accelerometer (B & K 4341) was bonded beneath the tip of the beam (using Eastman 910 adhesive). "Target" points were bonded to the upper surface of the beam at the four points shown in Figure 30.

The accelerometer's output was conditioned by a PCB* in-line amplifier with a voltage gain of ten. This instrumentation yielded a *total sensitivity of 30.3 mv/g.*

The beam was excited by striking the targets with an 8 oz. hammer as shown in Figure 31. A force transducer (PCB 208A03) was secured to the machined face of the hammer with a threaded stud. A plastic tip was mounted to the force transducer to serve as the striking surface. The transducer provided a *12.5 mv/lb sensitivity,* low impedance output.

The data was analyzed over a 2 kHz range. The 400A captured two simultaneous transients (force and acceleration) each time the beam was struck. The inertance function was *automatically computed* and displayed. The beam was struck 16 times at each target and the results averaged. *Resonant frequencies and amplitudes were read directly from the 400A's digital display.*

*PCB Piezotronics, Inc.
P.O. Box 33
Buffalo, N. Y. 14225

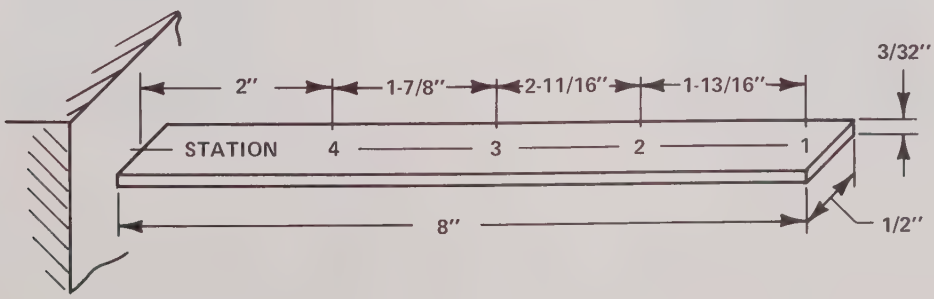


Figure 30: Cantilever Beam with 4 Measuring Points

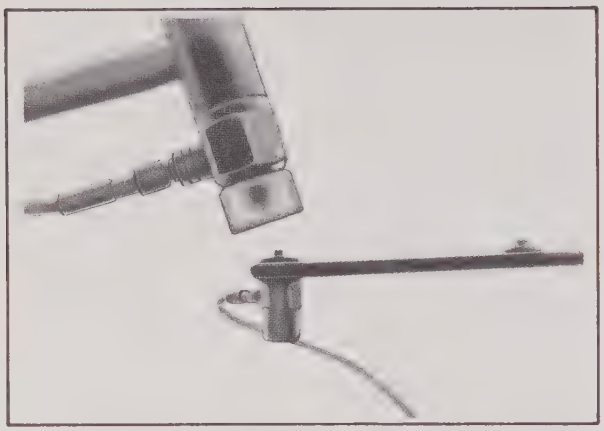


Figure 31: Hammer with Force Transducer Striking Beam at Station 1 (Accelerometer Mounted under Station 1)

Averaging over sixteen blows provided stable data with *coherence values in excess of 0.9 around each resonance.* This was an indication of *valid data,* free from nonlinear effects or noise contamination.

STEP 1: FINDING THE RESONANT FREQUENCIES

The beam was struck at the tip location (station I). The transfer function between force input and acceleration output (inertance) was displayed in real (Figure 32) and imaginary (Figure 33) form.

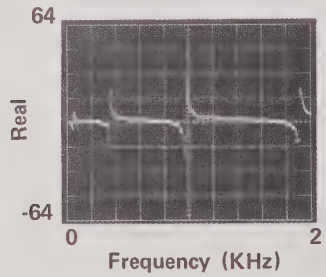


Figure 32: Driving Point Inertance (Station 1)

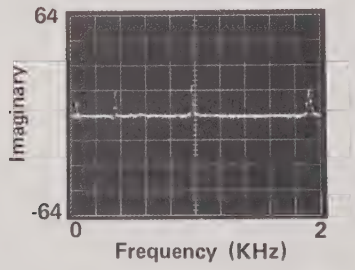


Figure 33: Driving Point Inertance (Station 1)

The resonant frequencies were read directly from Figure 33 using the cursor and digital readout of the 400A. The resonances corresponded with the peaks in Figure 33 and occurred at:

Mode (n)	Frequency (f_n)
1	50 Hz
2	350 Hz
3	970 Hz
4	1910 Hz

STEP 2: MEASURING THE DAMPING

Damping was measured from the frequencies of the maximum and minimum points shown in Figure 32 using method three. The results are summarized below:

Mode (m)	f_a (Hz)	f_b (Hz)	$Q = \frac{(f_a/f_b)^2 + 1}{(f_a/f_b)^2 - 1}$
1	60	50	5.545
2	350	340	34.507
3	980	970	97.503
4	1920	1900	95.503

STEP 3: MEASURING THE MODE SHAPES

The mode shapes were determined by *measuring the amplitude of imaginary inertance at each resonance for all four excitation positions.* The driving point function (Figure 33) was measured first, with the results:

Mode (n)	1	2	3	4
$IMAG_{b,b}$ (f_n)	7.93	15.1	19.5	17.0

Next, the hammer was used to excite the second location (a = 2), while the accelerometer remained fixed at the first location (b = 1). The amplitude of the imaginary response (Figure 34) was measured at the four resonant frequencies, f_n. The hammer was moved to the third and fourth locations (Figures 35 and 36) and these same measurements were made. Table 2 summarizes the results.

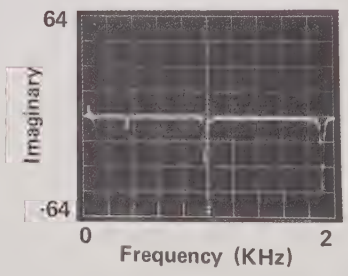


Figure 34: Transfer Inertance Between Stations 1 and 2

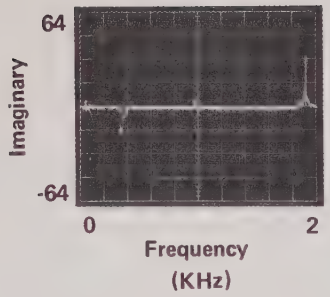


Figure 35: Transfer Inertance Between Stations 1 and 3

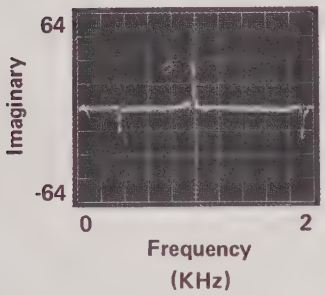


Figure 36: Transfer Inertance Between Stations 1 and 4

TABLE 2

Station Impacted (a)	Mode (n)			
	1	2	3	4
1	(7.93)	15.1	19.5	17.0
2	6.25	-7.31	-28.6	(-45.0)
3	3.68	(-17.5)	5.81	32.4
4	1.37	-16.0	(32.5)	-20.1
	$IMAG_{a,b}(f_n)$			

Each column in Table 2 is a mode shape vector. *The vectors are normalized by dividing each column by the largest number in it (number in parentheses).* The normalized mode shapes are listed in Table 3 and plotted in Figure 37.

TABLE 3

Station (a)	Mode (n)			
	1	2	3	4
1	1.00	-.863	.600	-.378
2	.7888	.418	-.880	1.000
3	.464	1.000	.179	-.720
4	.173	.914	1.000	.447
	a,n			

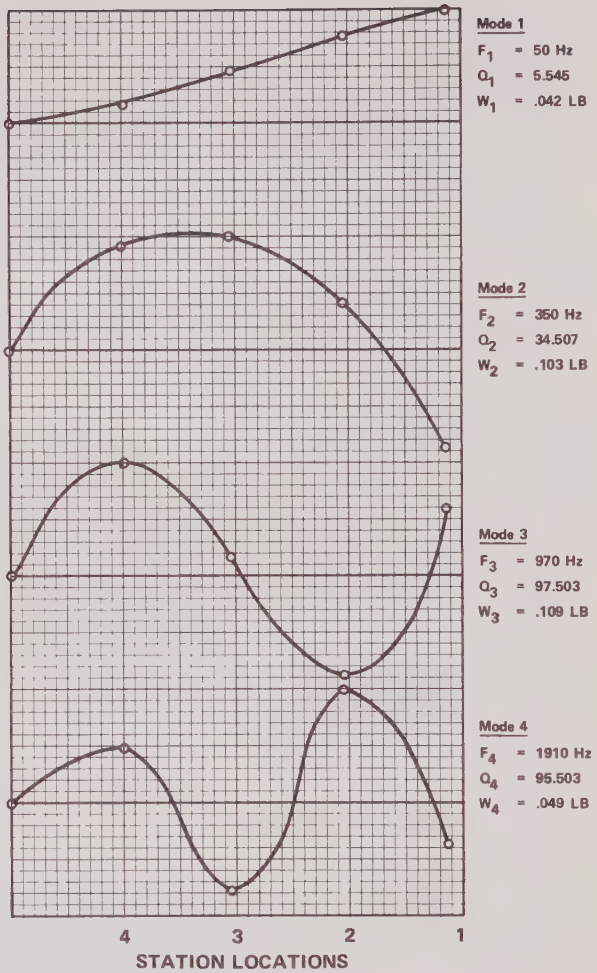


Figure 37: First Four Mode Shapes Measured from Cantilever Beam

STEP 4: CALCULATING THE MODAL WEIGHTS

Modal weights are computed from the data in Table 2. The calculation involves the numbers in parenthesis and the first row of numbers. Specifically:

n	$IMAG_{b,b}(f_n)$	$IMAG_{p,b}(f_n)$	$W_n = KQ_n \frac{IMAG_{b,b}(f_n)}{IMAG^2_{p,b}(f_n)}$
1	7.93	7.93	.042
2	15.1	-17.5	.103
3	19.5	32.5	.109
4	17.0	-45.0	.049

The constant, K, is evaluated from the sensitivity of the transducers and the selected input voltage ranges of the FFT analyzer. In this example, the force channel had a selected range of 40 volts (PTP) and the acceleration channel I volt (PTP). Specifically:

$$K = \frac{-S_b F_a}{S_a F_b} = \frac{-(-30.3 \text{ mv/g} \cdot 1 \text{ volt})}{(12.5 \text{ mv/lb} \cdot 40 \text{ volt})} = .0606 \text{ lb/g}$$

Note that the accelerometer sensitivity, S_b, was negative (in this example) because the accelerometer was *inverted* physically.

THE OMNIFEROUS OF-400A FFT ANALYZER

All measurements illustrated in this paper were made with the OF-400A shown in Figure 38. Ease of operation is a particular feature of this instrument-type FFT analyzer. Two channels of input data are processed simultaneously in real-time on the 10 kHz analysis range (over 25,000 samples per second on each channel). Employing extra-sharp built-in anti-aliasing filters, closely phase-matched for accurate two-channel analysis, the OF-400A produces 800 valid frequency points from its basic 2048-sample transforms.

Pushbutton-selected functions include: (a) two-channel spectrum analysis and power spectral density (averaged power spectrum), (b) cross-spectrum analysis, (c) transfer function, (d) coherence, (e) auto- and cross-correlation, and (f) dual-channel time-signal averaging. All frequency functions are calculated on-line from the same input data at the same speed, and can be viewed at any time during or after processing. No programming setup experience is required. Frequency ranges and functions are selected directly by simple pushbutton controls. An add-on mini-computer option allows further processing of calculated functions. Because the computer is not used in the basic processor, its memory is completely free for post-processing.

Features which make this powerful processor extremely simple to operate include:

- Complete compatability with impulsive, random and swept-sine test techniques.
- Dynamic range greater than 72 dB minimizes input attenuator readjustment and allows small signals to be measured at the same time as large ones.
- Digital readout of amplitude and frequency, both absolute or relative to peaks or frequency locations in the function itself, . . . reads in correct units (degrees, dB, %FS, Hz, etc.).
- Input range pushbuttons select both the correct sampling rate and the dual anti-aliasing filters in easily read 1-2-5 steps, eliminating the possibility of aliasing errors.
- Harmonic markers identify all peaks which are harmonically related to a selected fundamental.
- Line-and-dot cursor provides clear marking of both oscilloscopes for optimum readout of narrow-band peaks or random data . . . also marks frequency points on Nyquist displays.
- Two display outputs are provided, each with separately set calibrated scales for linear, log, and phase . . . eliminates readjustment and allows direct measurements.
- Complete provision for recording or plotting of any function.

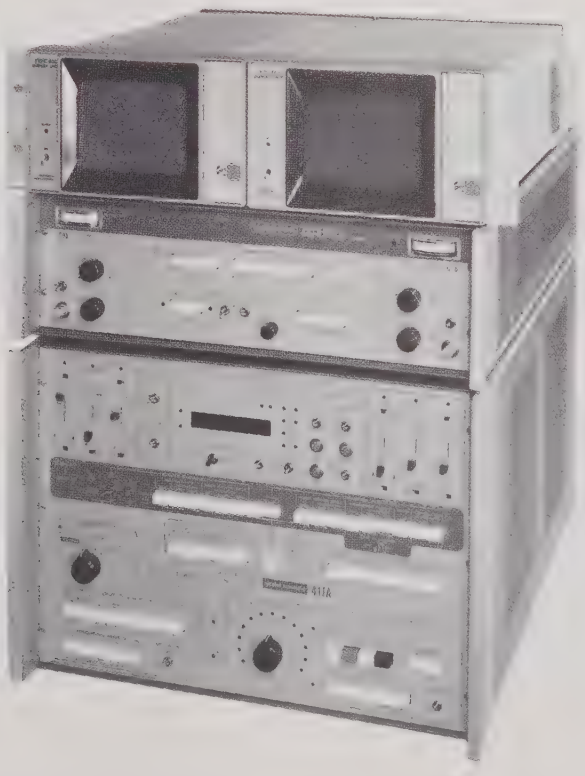


Figure 38: Model OF-400A Omniferous™ FFT Analyzer

SUMMARY OF FREQUENCY FUNCTIONS

FREQUENCY FUNCTIONS

DEFINITION OF FUNCTIONS COMPUTED

Function Computed	Symbol	Equation	Description
Fourier Transform	S_A	$= \mathcal{F}\{A(t)\} = \int_0^\infty A(t)e^{-2\pi ft}dt$ $= \lvert S_A\rvert \cos(\phi_A) + j\lvert S_A\rvert \sin(\phi_A)$	Complex spectrum of time history, A(t). Magnitude $\lvert S_A\rvert$ defines instantaneous spectral content. Phase angle, ϕ_A, describes *relative phasing* of periodic harmonic components. Absolute angles are only meaningful if analysis window is synchronized to periodic harmonic data.
Averager Power Spectrum	G_{AA}	$= \overline{S_A \cdot S_A}^*$ $= \overline{S_A}^2$	Ensemble average of squared spectrum magnitude (formed by multiplying complex spectrum by its conjugate). It is a real spectrum, computed at each frequency across a series of instantaneous spectra, a frequency domain statistic defining *average power* (variance) at each frequency.
Average Cross Spectrum	G_{AB}	$= \overline{S_B \cdot S_A}^*$ $= \lvert\overline{S_A}\rvert\lvert\overline{S_B}\rvert \cos(\overline{\phi_B - \phi_A})$ $+j\lvert\overline{S_A}\rvert\lvert\overline{S_B}\rvert \sin(\overline{\phi_B - \phi_A})$	Ensemble averaged complex product of spectrum, S_B, with conjugated spectrum, $S_A{}^*$. Indicates those frequencies where *both* A(t) and B(t) have content. Phase angles indicate average phase *difference* between signals at each frequency.
Transfer Function	H_{AB}	$= \frac{G_{AB}}{G_{AA}}$ $= \frac{\lvert G_{AB}\rvert}{\lvert G_{AA}\rvert} \cos(\overline{\phi_B - \phi_A})$ $+ j\frac{\lvert G_{AB}\rvert}{\lvert G_{AA}\rvert} \sin(\overline{\phi_B - \phi_A})$	Complex ratio of cross spectrum, G_{AB}, to input power spectrum, G_{AA}, defines the gain and phase lag introduced by a process *excited* by A(t) and *responding* with B(t).
Coherence Function	γ_{AB}^2	$= \frac{G_{AB} \cdot G_{AB}^*}{G_{AA} \cdot G_{BB}}$ $= \frac{\lvert G_{AB}\rvert^2}{G_{AA} \cdot G_{BB}}$	Real Spectrum "power squared" ratio indicates *linear cause/effect* relationship between A(t) and B(t). Linear process excited *only* by A(t) yields $\gamma_{AB}^2 = 1$ at all frequencies. Non-linearities or unmeasured inputs produce γ_{AB}^2 between "0" and "1".

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A P P E N D I X G

(Follows)

APPLICATION NOTE 10
OCTOBER 1975

SHAKE, RATTLE, OR RAP?

OR HOW TO CONDUCT VIBRATION TESTS

WRITTEN BY: GEORGE F. LANG
MANAGER APPLICATION ENGINEERING

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INTRODUCTION

Mechanical structures transfer force and motion, which by nature are inseparable. The transfer of one variable always accompanies the transfer of the other and in product they constitute mechanical energy. Structural mass, stiffness and damping characteristics all impede the transfer of forced dynamic motion. These structural characteristics tend to delay or distort the force or displacement quantity being transferred. Understanding the nature of this transfer process is fundamental to understanding the behavior of a structure under its environmental loading situation.

The transfer characteristics may be measured by isolating the structure from its normal environmental loading and subjecting it to an excitation of simpler nature. In practice, the structure is subjected to a force introduced at a single point with a fixed orientation and the resultant motions are measured at one or more fixed points. This permits measurement of a series of *transfer functions,* characterizing the structure.

Each *transfer function* provides two pieces of cause/effect information:

(1) The ratio of force/motion (as a function of frequency) relating the force input point and the motional response point.

(2) The degree of phase lag (as a function of frequency) between the force and motion introduced by the structural path.

Transfer functions identify* the resonant frequencies, damping characteristics, mode shapes, stiffness and inertia of the structure.

Six *transfer function* types are frequently measured. These are:

(1) Compliance = Displacement/Force
(2) Mobility = Velocity/Force
(3) Inertance = Acceleration/Force

(4) Apparent Stiffness = Force/Displacement
(5) Impedance = Force/Velocity
(6) Apparent Mass = Force/Acceleration

Until the mid 1960's, *transfer functions* were measured by carefully controlling the amplitude of excitation to be constant and measuring the resultant response. That is, the denominator term was held constant (for a range of frequencies) and the numerator was measured (within the controlled frequency range). More recently, dual channel Fast Fourier Transform (FFT) instruments permit simultaneous measurement of both terms, eliminating the need for carefully controlled inputs.

* See Nicolet Scientific Corporation's Application Note 9, titled:
"Understanding Vibration Measurements"

THREE TEST PROCEDURES

The transfer functions define the sinusoidal input/output relationship of a structure but they can be measured using non-sinusoidal excitation. In fact, three broadly different test procedures are currently used. These techniques are called swept sine, random and impulsive excitation. Each has certain inherent advantages and disadvantages. There is no "right" technique for all problems. Fortunately, modern measurement equipment (such as the Nicolet Scientific OF-400A) are fully capable of performing analysis for all three types of testing.

SWEPT SINE TESTING

Swept sine testing is the oldest of the three techniques. In this procedure, the structure is subjected to sinusoidal excitation and the frequency of the sinusoid is slowly swept through a range of frequencies. If the sweep rate is sufficiently slow (as determined both by the structure and the measurement equipment) the transfer functions computed from this technique are identical to the results obtained by "resting" at a sequence of frequencies. Figure 1 illustrates a complete swept sine analysis system capable of measuring any of the six frequency response functions.

The swept sine analysis system illustrated in Figure 1 produces transfer functions directly on an analog X-Y plotter. The plotter's X axis is driven by a DC voltage proportional to the excitation frequency while the Y axis of the plotter is driven by the detected output of a filter monitoring the response of the structure at the excitation frequency. The excitation sinusoid is generated by an oscillator whose frequency can be altered by a sweep generator.

In older equipment the sweep generation is performed by an electrical motor rotating a variable resistor in the oscillator circuit. In more modern equipment, the oscillator is voltage controlled so that the sweep is generated by a voltage proportional to the desired frequency. Most modern sweep generators are capable of operating with either linear or logarithmic sweep in an ascending, descending or alternating pattern.

The output of the oscillator passes through two control circuits (the mass compensator and amplitude compressor) to a power amplifier. The amplifier drives the electrical coil of an electrodynamic shaker or the control valve of a hydraulic shaker.

The test object is mechanically attached to the shaker through a transducer set called an impedance head (though inertance head might be a more proper name). The impedance head contains an accelerometer and a force transducer. Attachment between the test object and the impedance head is normally made with a short rod structure called a "quill shaft."

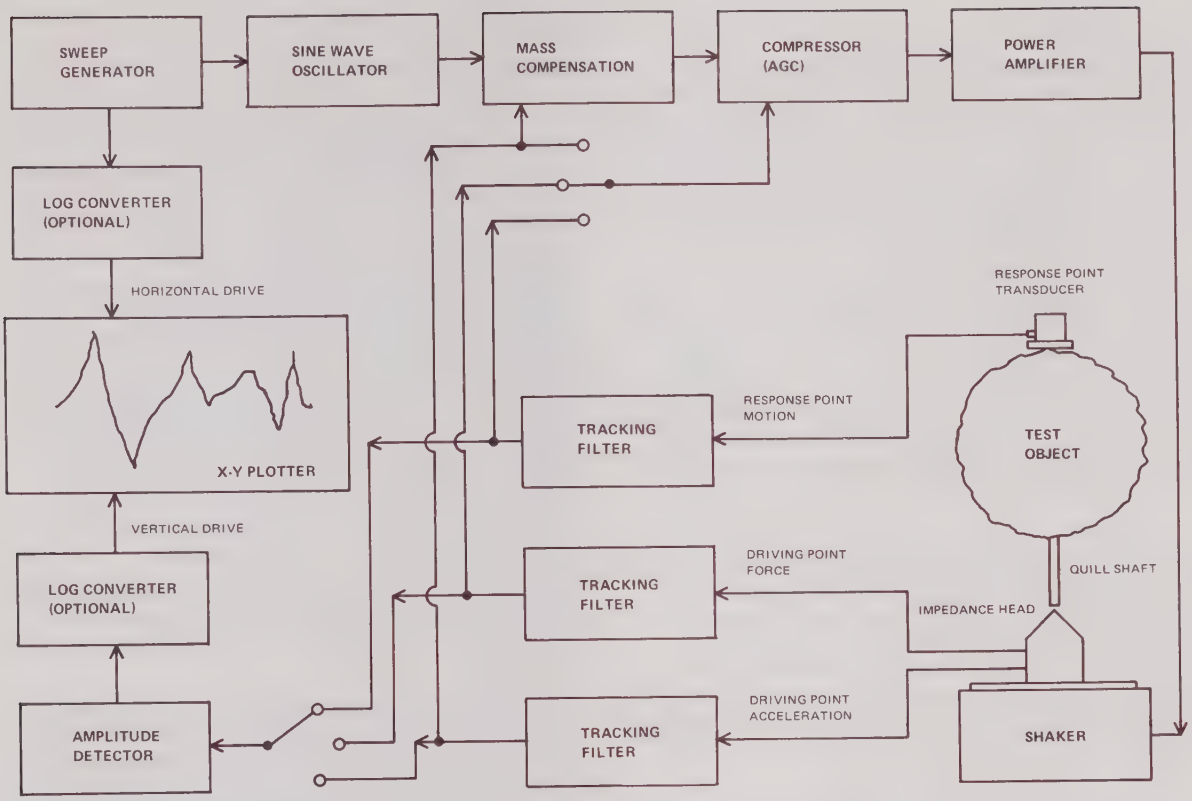

Figure 1. Typical Swept-Sine Analysis System

When spacial transfer measurements are being made, an additional transducer (accelerometer, velocity pickup, or displacement indicator) is placed at the response point on the test object. All three transducer outputs are monitored by tracking filters. The tracking filters are tuned to the frequency of the oscillator and vary their center frequency as the oscillator sweeps.

The compressor circuit is an automatic gain control used to maintain a constant amplitude excitation or response. This circuit may be used in conjunction with any of the three transducers. The compressor circuit closes a feedback loop around the shaker and power amplifier by measuring the selected feedback variable (driving point force, driving point acceleration, or response point motion) comparing it with a reference, and adjusting the system gain upwards or downwards to maintain a steady state amplitude. This circuit functions to compensate the excitation system for changes in impedance caused by the dynamics of the test object.

When measuring compliance, mobility or inertance (either driving point or transfer) driving point force feedback is employed. When measuring apparent stiffness, displacement feedback is employed. When measuring impedance velocity feedback is employed. Apparent mass is measured by using acceleration feedback.

When driving point stiffness, impedance or mass are being measured, the feedback signal is derived from the driving point accelerometer. Electronic integration is used, if required, to obtain a velocity or displacement feedback. When transfer characteristics between two locations are being measured, the feedback signal is derived from the response point transducer.

The mass compensation circuit is necessary when conducting experiments on structures where the attachment point is relatively light weight. This circuit functions to correct the driving point force measured by the driving point force transducer for the force required to accelerate the quill shaft. The mass compensation circuit corrects the force by subtracting a portion of the driving point acceleration signal from the driving point force signal.

The mass compensation is calibrated prior to attaching the shaker to the test object. This is accomplished by placing a known mass on the quill shaft (of approximately the same weight as the force levels to be used during the examination of the structure) and exciting the mass using the shaker system. The mass compensation network is adjusted until the apparent mass measured in this calibration test equals the actual calibration mass placed on the quill shaft. Thereafter, the mass compensation circuit continuously corrects the output of the driving point force transducer so that it is true measure of the force placed into the test object.

The response variable (Y axis) is derived by amplitude detecting the output of one of the three tracking filters. Additional circuitry may be placed between the amplitude detector and the plotter to perform logarithmic conversion of the amplitude detector's output. Likewise, logarithmic conversion may be performed on the X axis signal generated by the sweep generator.

When data are plotted in log/log format, synthetic integration (or differentiation) may be employed. This procedure allows transfer functions to be presented as though the response variable had been integrated (or differentiated). Thus an inertance function (acceleration/force) could be double integrated to produce a compliance (displacement/force) plot. Synthetic integration is accomplished by electronically subtracting a slope from the data being plotted.

Optionally, the characteristics of the amplitude detector may be modified (using the output of the oscillator) to provide the coincident (real) or quadrature (imaginary) portion of the response variable. In some systems, two amplitude detectors are applied to the output of the tracking filter, one producing the coincident response and the other producing the quadrature response. Such systems are called co/quad analyzers.

ADVANTAGES OF SWEPT SINE

Swept sine testing has a number of distinct advantages.

1. It is the most precise means of exciting the structure.
2. It provides flexibility in defining amplitude dependent non-linearities by repeating experiments at different excitation amplitude levels.
3. Swept sine systems are capable of placing high excitation powers into examined structure. This is advantageous in defining the properties of large or highly damped structures.

In general, it is possible to put more power in at low frequencies using swept analysis than any other technique. Techniques may be used to enhance the dynamic range of excitation (and therefore the signal-to-noise ratio of measurements) by changing the nature of the feedback variable during the course of a frequency sweep. For example, the feedback variable may be changed from displacement to velocity to acceleration during the course of the sweep. Alternatively, control may be switched between the excitation and response variables when examining systems with very high "Q" resonances. Swept sine analysis provides continuously variable resolution and is thus particularly useful for defining high "Q" modes.

DISADVANTAGES OF SWEPT SINE

1. The rapidity with which the sinusoidal frequency can be swept is determined by the bandwidth (or Q) of the modes to be defined *and the resolution or bandwidth of the analysis filter.* In general, it is necessary to use analysis filters with narrower bandwidth (higher Q) than any of the modes to be defined. This constraint causes the limiting factor of the sweep rate to be dictated by the analysis filter. If the analysis is conducted with a filter of noise bandwidth, β, the frequency can be swept no faster than β^2 Hz per second. This restriction is imposed because of the settling time of the analysis filter. Sweeping in excess of β^2 Hz per second will introduce substantial transient response from the filter which will manifest itself as an apparent "broadening" of the filter's bandwidth. Thus if a system with low frequency modes of high "Q" is being examined, the sweep rates must be extremely slow, *imposing long test times.*

2. Secondly, swept analysis *only functions for stationary situations.* That is, if the system under examination has time variant parameters, this technique cannot be employed.

3. Swept sine analysis is *predicated on the mechanical attachment of a drive shaker and a considerable amount of set-up time is involved for each excitation point.* Analysis of very high frequency systems imposes an additional restriction. The nature of the quill shaft used to connect the structure to the shaker may become a significant portion of the dynamic system. In this situation, the entire concept breaks down and is impossible to analyze the structure independent of the excitation system.

4. The implementation of a swept sine system *uses a large amount of analog equipment with a need for frequent calibration and check-up.* Analog equipment, in general, suffers from more down time and calibration need than equivalent digital circuitry.

5. Swept sine excitation does not simulate the operating environment of most systems. In practice, many vibration frequencies co-exist and may interact.

RANDOM EXCITATION TESTING

Random testing can be accomplished in one of two fashions. The first of these is a true random excitation as illustrated in Figure 2. The source of excitation is provided by a broad band noise generator which produces an output signal with a "white" spectrum (a spectrum flat to at least the bandwidth of analysis). Probability distribution of the broad band excitation is Gausian. This source of excitation is fed through a mass compensation network (if required, for the examination of light structures with high "Q" modes) then through a power amplifier to the drive coil of the shaker. Both the excitation force and the response variable are measured simultaneously using a dual channel Fast Fourier Transform processor (such as the Nicolet Scientific OF-400A).

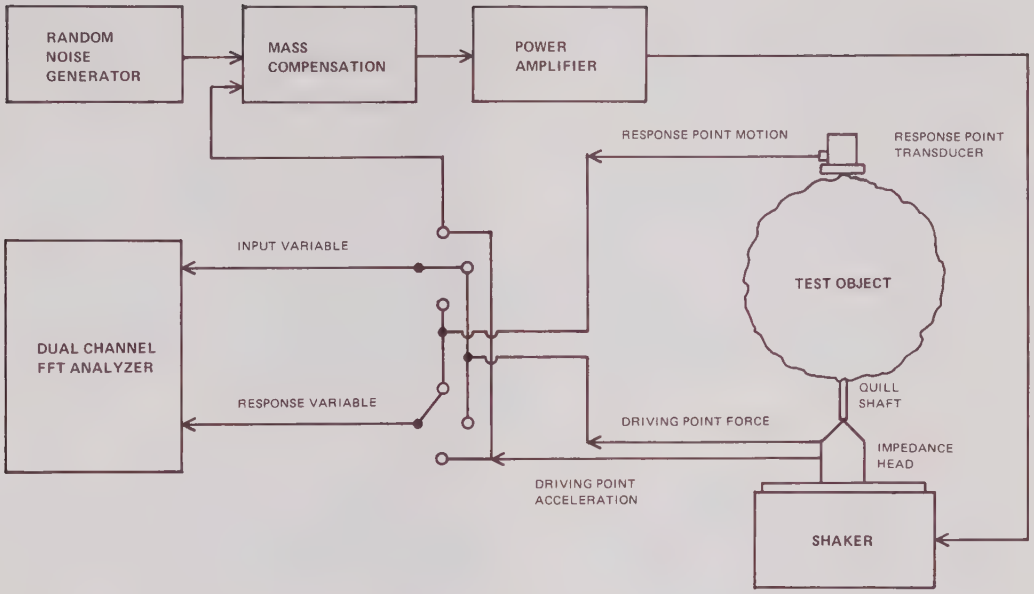

Figure 2. Typical Random Excitation System

The broad band excitation excites all frequencies simultaneously and thus data at all frequencies is immediately available and may be viewed on an oscilloscope. To enhance the statistical confidence in the measure, it is necessary to perform some averaging from the resultant spectra. This is accomplished by averaging the power spectrum of the input variable and the cross spectrum between the input and the response variables. The transfer function is computed by performing a frequency point by frequency point division of the complex cross spectrum by the power spectrum. This technique takes full advantage of the "coherent" averaging process.

The random spectrum need not be exactly "white." Both the input power spectrum and the cross spectrum reflect the average content of the excitation, hence deviations from a "white" input are cancelled out. Furthermore, relatively little averaging is required to compute the transfer function as random fluctuations are common to both the numerator and denominator.

When examining systems with high "Q" modes, the random spectrum may be "shaped" to reduce excitation around the resonances and increase it around the nodes. This procedure vastly improves the signal-to-noise ratio of the measurement by compressing the necessary dynamic range of the analyzer.

A variation on the random testing scheme is the use of pseudo random noise. Pseudo random testing may be conducted slowly using a signal constructed by the FFT analyzer by inverse transforming a desired spectral function. More efficient and flexible systems utilize a pseudo-random noise generator, sequence locked to the FFT analyzer.

ADVANTAGES OF RANDOM TESTING

1. *Random testing excites all frequencies simultaneously.* This permits the investigation of structures with time varying characteristics. Using exponential averaging in the computation of the power and cross spectra permits on-line observation of these changes. This is particularly valuable when "tuning" a structure by adding mass, stiffness or damping.

2. This excitation techniques is *significantly faster than the application of swept sine* analysis. Trade-offs between frequency resolution, statistical confidence and testing time may be readily made. Because both the input and response variables are measured, the required resolution bandwidth may be selected to match the structural characteristics of the test object.

3. *Random excitation "exercises" any amplitude-dependent non-linearities in the structure.* The resultant transfer functions indicate behavior averaged over these non-linearities. If required, the coherence function (cross spectrum squared divided by input and response power spectra) may be displayed to characterize the amount and frequency region of non-linear response.

DISADVANTAGES OF RANDOM TESTING

Random techniques have some inherent disadvantages:

1. They may require a larger shaker system than swept sine analysis to excite the same structure. In many situations, the random testing procedure will be power limited as it is not always possible to excite large, heavy structures over a broad range of frequencies with available excitors.

2. Additionally, random excitation produces poorer signal to noise ratios than swep sine analysis, although the disadvantage may be minimized by increasing the averaging period.

3. Random techniques require more expensive instrumentation than the swept sine equipment. It is still necessary to set up shaker equipment and when working with light weight, lightly damped systems mass cancellation techniques are still required.

IMPULSIVE TESTING

The third class of frequency response excitation is the "impulsive force input" technique. This procedure utilizes short duration transient force inputs (with corresponding broad band spectra) to excite all frequencies in the structure simultaneously. Impulsive testing is normally conducted using some form of an instrumented hammer such as that shown in Figure 3. A force transducer is mounted either to the head of the hammer or to that portion of the structure to be impacted.

An accelerometer is used to measure the response of the structure. By far the most popular technique for impulsive excitation is to mount an accelerometer at a fixed location and to excite the structure at a multiplicity of locations using a hammer with a force transducer on its face to impulse the system at many locations.

The data is analyzed with a dual channel Fast Fourier Transform analyzer. Each time the structure is impulsed with the hammer, both the impulse and the response acceleration are captured by the FFT analyzer using the impulse force as a transient capture trigger condition. These two transient time histories are Fourier transformed to yield the input and output spectra. The resultant ratio of the output and input spectra is the desired transfer function.

Usually, the results of several transient excitations are averaged. In this situation, the most desirable procedure is to compute the cross spectrum between the input and response and the power spectrum of the input. The resultant averaged transfer characteristic is derived by dividing the cross spectrum by the input power spectrum.

ADVANTAGES OF IMPULSIVE TESTING

Impulse testing is, by far, *the most rapid technique* that can be applied to the study of a complex structure. Its fundamental advantage is that no shaker system needs to be mounted to the test object. The testing time required for each spacial position on the structure is determined only by the number of times the tester wishes to impact the structure with a hammer. Because the response transducer is mounted at a fixed location, no time is required to move the response transducer from point to point and, of course, a hammer may be easily moved from one point on the structure to another.

DISADVANTAGES OF IMPULSIVE TESTING

1. Impulsive techniques impose stringent requirements on the analysis instrumentation. Specifically, the FFT analyzer must have considerable more dynamic range than is required for sinusoidal or random testing.

2. Precision of the results is strongly affected by the care with which data is initially acquired. If the signal conditioning amplifier or analog to digital converters of the FFT analyzer are allowed to momentarily overload during the acquisition of a transient, the results are grossly affected.

3. It is important to match the duration of the impulsive force input to the band of frequencies to be analyzed. This requires selecting the mass and geometry of the hammer as well as the nature of the material at the hammer/structure interface to "shape" the input pulse.

4. The mass of the striking "tip" between the force transducer (mounted on the face of the hammer) and the structure can introduce mass errors when exciting light structures. Unfortunately, there is no currently available technique for performing on-line mass cancellation (but the need for such correction is less than that required for any other technique). The results of impulsive testing have lower signal to noise ratio than those resulting from any other form of excitation.

SUMMARY

In general, sinusoidal testing is the slowest but most precise procedure for performing frequency response measurements. Impulsive force input techniques are the fastest but least precise. The random technique lies in the middle ground. It is particularly useful for examining on-line changes in a structure while trial "fixes" are adjusted or "tuned."

Sinusoidal techniques have retained their popularity through history. Transient techniques have come into recent vogue as a means of performing rapid analysis on complex problems. The quick "look-see" capability offered by impulsive techniques is often sufficient to solve a problem. Frequently initial studies are conducted using impulsive techniques and higher resolution infomation is gathered (where required) using either the random or swept sine techniques.

FFT processing analyzers have gained enormous popularity because they are capable of performing transfer function measurements using any of the three types of excitation. Hence, there may be no universal "proper" choice of excitation technique for testing but there is a clear choice for attendant instrumentation. Figure 4 illustrates such a choice!

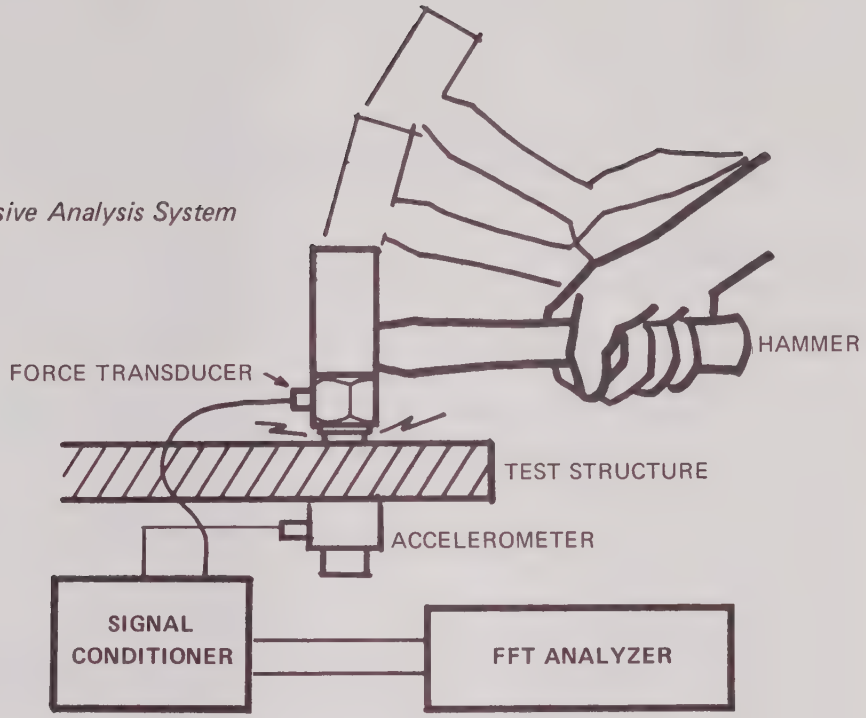

Figure 3. Typical Impulsive Analysis System

THE OMNIFEROUS OF-400A FFT ANALYZER

Features which make this powerful processor extremely simple to operate include:

- Complete compatibility with swept-sine, random and impulsive test techniques.
- Dynamic range greater than 72 dB minimizes input attenuator readjustment and allows small signals to be measured at the same time as large ones.
- Digital readout of amplitude and frequency, both absolute or relative to peaks or frequency locations in the function itself, . . . reads in correct units (degrees, dB, %FS, Hz, etc.).
- Input range pushbuttons select both the correct sampling rate and the dual anti-aliasing filters in easily read 1-2-5 steps, eliminating the possibility of aliasing errors.
- Harmonic markers identify all peaks which are harmonically related to a selected fundamental.
- Line-and-dot cursor provides clear marking of both oscilloscopes for optimum readout of narrow-band peaks or random data . . . also marks frequency points on Nyquist displays.
- Two display outputs are provided, each with separately set calibrated scales for linear, log, and phase . . . eliminates readjustment and allows direct measurements.
- Complete provision for recording or plotting of any function.

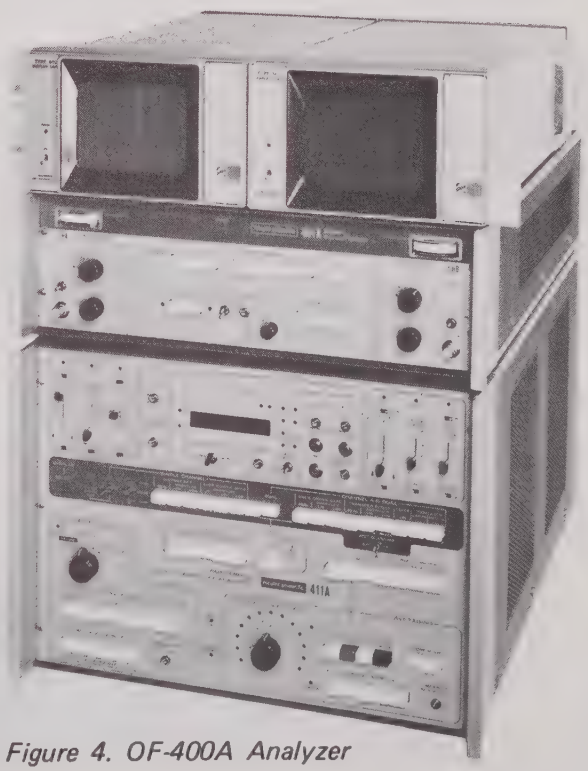

Figure 4. OF-400A Analyzer

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A P P E N D I X H

(Follows)

Preliminary Note
June 1975

DUAL CHANNEL ANALYSIS

ANSWERS THREE MAJOR QUESTIONS

Transfer, Coherence and Coherent Output Power Functions Explained

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DUAL CHANNEL ANALYSIS ANSWERS THREE MAJOR QUESTIONS

Experimental measurements are frequently made to answer one (or more) of three fundamental questions. Simplistically, these questions may be stated: "what's in the box?", "who did that?", and "where did I go wrong?". Interestingly enough, although the questions are quite different the means of answering them are markedly similar.

Figure 1 illustrates the "what's in the box?" question. In this situation it is necessary to characterize a process from knowledge of its input and output. The process to be characterized might be electrical, mechanical, acoustical, or physiological in nature. The experimenter is faced with the situation of being able to measure the input or stimulus to the process and the output or response of the process to that stimulus. To the experimenter the process represents a "black box" whose workings are to be understood. The input to the system, X(t), and the output of the system, Y(t), are time histories that can be measured.

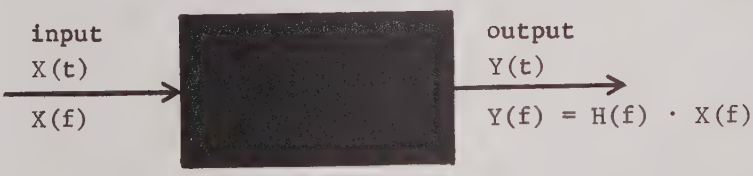


FIGURE 1: "WHAT'S IN THE BOX?"

THE TRANSFER FUNCTION

The powerful technique of Fourier analysis can be used to characterize "what's in the box." This is accomplished by computing a transfer function between the signals X and Y. The transfer function is a frequency domain description of how the box operates on X to produce Y. It contains two pieces of information:

1. The ratio of Y/X as a function of frequency.

2. The phase lag between Y and X as a function of frequency.

The transfer function is the ratio of the spectra associated with the signals X and Y. It is computed by Fourier transforming the time histories X(t) and Y(t) to yield the instantaneous spectra X(f) and Y(f). The complex ratio Y(f)/X(f) is a measured estimate of the transfer function. Such a measurement is, of course, a statistical estimate of the true transfer function. As with any estimate the measurement may be improved by averaging. A significant improvement in the statistics of a transfer function measurement can be achieved by computing the transfer function estimate as the ratio of the complex cross spectrum (relating X and Y) to the power spectrum of X. The cross spectrum is computed by averaging the complex product of the spectrum of Y with the conjugate of the spectrum of X. The power spectrum of X is computed by averaging the squared magnitude of the spectrum of X.

Knowledge of the transfer function answers our first question as illustrated in Figure 2. That is, knowing the transfer function, H(f) of the process allows prediction of the response of the system, Y to any given input X. The simple multiplicative nature of the relationship makes the transfer function a simple means of characterizing the process. (Attempting to characterize the system in the time domain is a far more complicated business requiring the use of convolution integrals.)

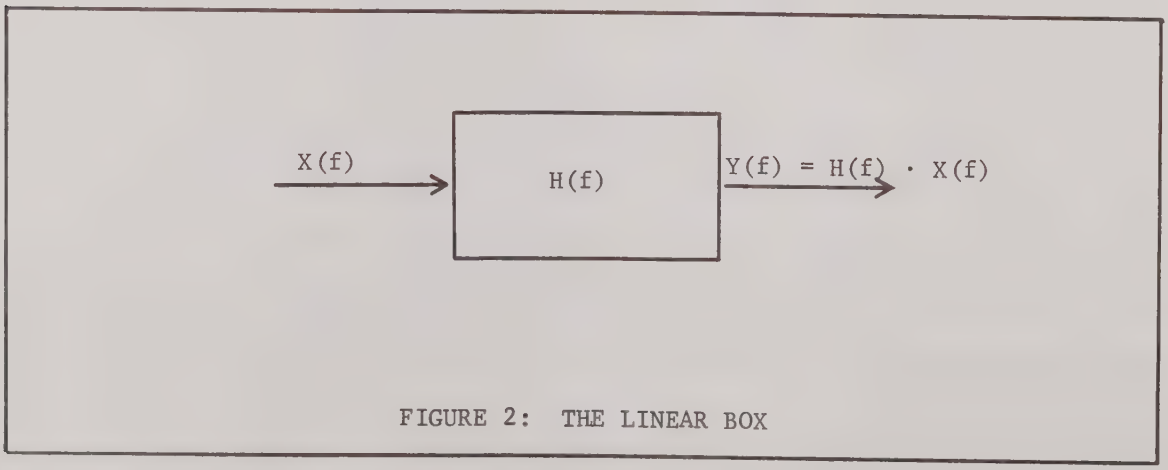


FIGURE 2: THE LINEAR BOX

The notion of a transfer function assumes that the process to be characterized is linear in nature. Fortunately, a broad range "real world" phenomena are (at least to first order effect) linear. If the "box" can be showed to be predominantly linear the transfer function is a valid means of characterizing it. Unfortunately, this is not always true.

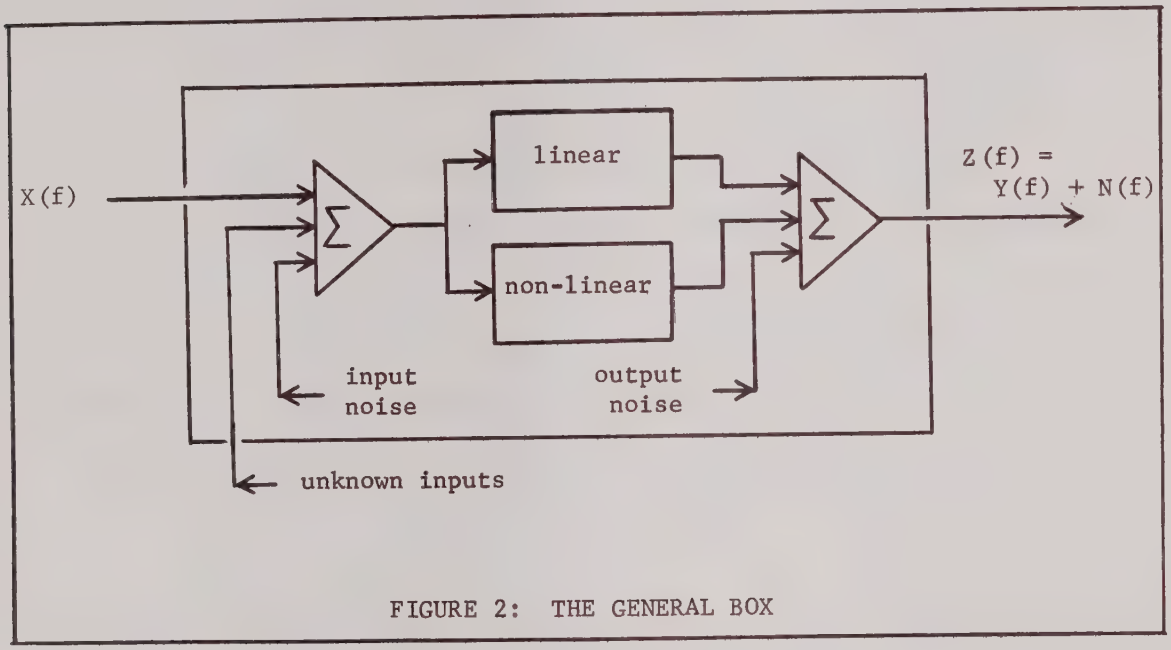


FIGURE 2: THE GENERAL BOX

THE COHERENCE FUNCTION

Figure 3 illustrates a more general concept of what might be in the "box." The "box" may have two parallel processes, one of which is linear (and therefore may be characterized by a transfer function) and a second which is non-linear (and cannot be characterized by a transfer function). The input to this system may consist of the measured stimulus X and a summation of other terms. The other terms can include unmeasured stimuli (from inputs unknown to the experimenter) and input noise unrelated to either the linear or non-linear process. Additionally, the "box" may generate output noise unrelated to the internal processing path. Thus, the output available to the experimenter, Z, is actually a summation of terms. The spectrum of this output may be considered as the summation of two terms, that is:

$$Z(f) = Y(f) + N(f)$$

The spectrum Y(f) is due to the linear response of the system to the measured stimulus X. The spectrum N(f) represents a measure of contamination containing three distinct constituents:

1. The non-linear response of the process to the stimulus X.
2. Any response, linear or non-linear of the system to unmeasured stimuli and input noise.
3. Noise introduced at the output of the process including that generated by measuring apparatus.

In this situation the cross spectrum between X and Y (G_{xy}) cannot be measured directly. It is, however, possible to measure the cross spectrum between X and Z (G_{xz}) and the power spectra of X and Z (G_{xx} and G_{zz} respectively). From these three measurements considerable insight may be gained.

The coherence function, $\gamma^2(f)$, may be computed from the cross spectrum and power spectra. Specifically:

$$\gamma^2(f) = \frac{G_{xz}^2}{G_{xx} \cdot G_{yy}}$$

This "power squared" ratio can be proved (mathematically) equivalent to the statement:

$$\gamma^2(f) = \frac{G_{yy}}{G_{yy} + G_{nn}}$$

That is, the coherence function is the ratio of the power spectrum of Y (which cannot be measured physically) to the summation of the power spectra of Y and N.

If the process being evaluated was actually as simple as the "box" shown in Figure 2 then the noise power spectrum, G_{nn}, would be zero and the coherence function would be equal to 1 at all frequencies. As the degree of significance of the three forms of contamination increases, the coherence function asymptotically approaches zero. Thus, the coherence function expresses the degree of linear

causality between X and Z as a function of frequency. It can be used, therefore, to determine over what frequency range a transfer function is a valid description of "what's in the box." At those frequencies where the coherence function approaches 1 a measured transfer function between X and Z is a valid characterization of the process. In those frequency intervals where the coherence function approaches zero such a model is invalid.

The coherence function may be used, directly, to answer our second question "who did that?". Consider the situation illustrated by Figure 4. A single output (either desirable or problematic) is potentially caused by a summation of parallel processing paths. The problem here is to determine which path is influential in a particular frequency region. A series of coherence measurements made between the output and each of the potential causes can quickly determine the degree of influence of each path.

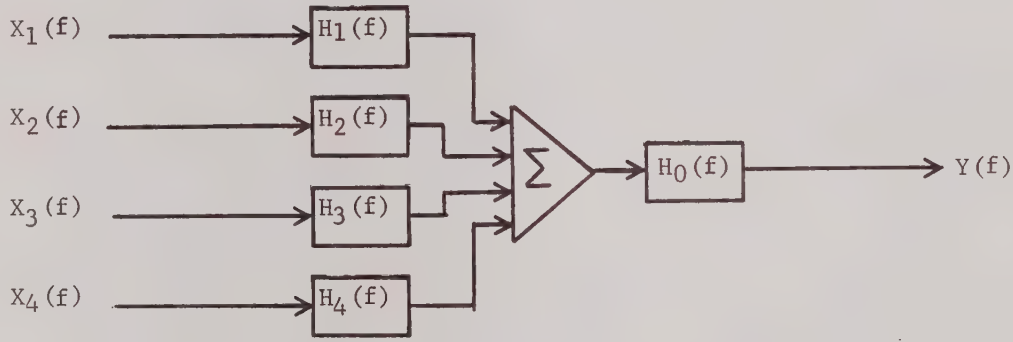


FIGURE 3: "WHO DID THAT?"

In this discussion the assumption is made that the probable causes all represent statistically independent events, that is, they are uncorrelated. Techniques exist for solving the more complicated situation where the inputs are related; but they are beyond the scope of this discussion. It is also assumed that

each of the parallel processing paths is characterized by predominantly linear behavior. In practice, these assumptions are not unduly restrictive as many actual problems involve linear behavior and statistically independent multiple stimuli.

Earlier approaches to solving this question involved looking for similarities between the effect and each probable cause. The degree of similarity was measured by computing time domain cross-correlations between the effect and suspected causes. In a frequency domain sense this is equivalent to measuring the cross spectra between the suspects and effect.

The correlation function is not as definative as the coherence function, because it attempts to simultaneously detect the causality and characterize both the process and the nature of the stimulus. Hence, the results of correlation are often quite confusing, while the coherence function is unambiguous. Correlation techniques are most useful when the stimulus is broad band. If the stimulus contains periodic components the correlation function is almost useless, as its results are strongly biased to the lowest frequency periodic component. The coherence function does not suffer this frailty.

The linear cause/effect detection provided by the coherence function has many practical applications. It has been used to determine at what point in a circuit undesirable noise is introduced. It has discerned which of several alternative excitations produces an undesirable vibration in a structure. Undesirable pressure pulsations in hydraulic systems have been traced to their major cause (among several possibilities) via coherence measurement. Predominant acoustic paths transmitting undesirable noise to a listener have been identified. Because of its normalized form, the coherence function offers an unambiguous answer to "who did that?".

The coherence function also answers our third question, "where did I go wrong?". Figure 5 illustrates this problem situation. In this case, the "box" is expected to be well understood and of linear behavior. Unfortunately, this is not always the case. When a coherence measurement between a designed system's input and output is low, indicating non-linear (or at least unanticipated) performance, the designer throws up his hands and asks our third question, "where did I go wrong?".

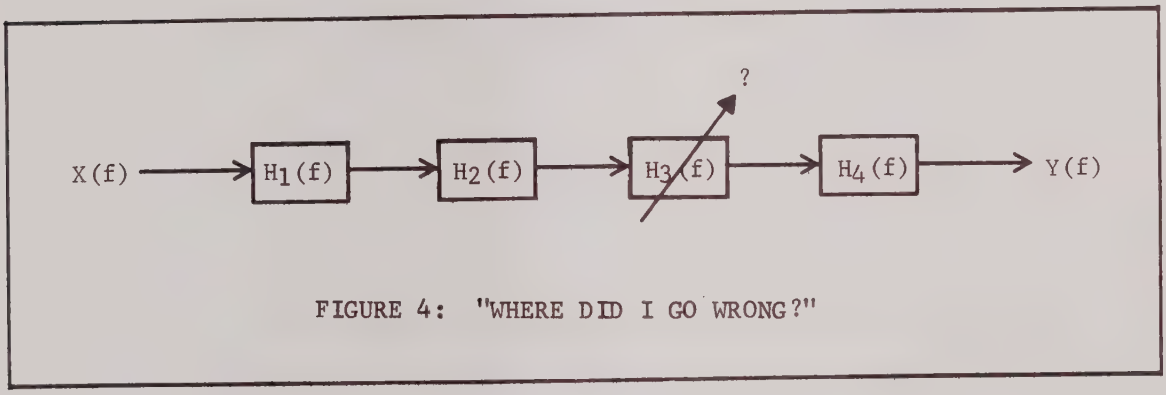


FIGURE 4: "WHERE DID I GO WRONG?"

The designer must now deduce which stage within the total process is exhibiting non-linear or undesirable behavior. This answer is achieved by making coherence measurements across each stage of the sequential process or, alternatively, making coherence measurements between sequentially cascaded stages of the process starting from one end or the other. Once a stage is found which exhibits a low coherence measurement within the problematic band of frequencies the first design failure has been located. In general, because it is possible to have multiple design failures it is most desirable to test the coherence across each stage of

the cascaded process. This allows simultaneous redevelopment of all faulty stages detected. Thus, the coherence function directly defines "where did I go wrong?".

THE COHERENT OUTPUT POWER

The rapid development of Fourier transform based instrumentation has given rise to an equally rapid development in the associated vocabulary. This meteoric increase in technical verbage provides a golden opportunity for confusion. The coherence function has a pale imitator. This function is called the "coherent output power" (COP). The coherent output power has a valid application in a very small subset of the problems solved by the coherence function. The coherent output power provides a modified answer to the question "who did that?". This modified answer is valuable if a combined question is stated: "who did that, and how much did he do?".

Consider the situation depicted by Figure 4. In this case, the experimenter is observing the summation of events caused by a series of independent stimuli. His desire is to record the power spectra associated with each event independently. Unfortunately, the nature of the environment precludes his making the measurements independently. In this situation his only viable alternative is to measure the total power of the summation of possible inputs and the coherence function between each of the stimuli and the total. Sequentially multiplying each coherence function by the total output power provides a series of "coherent output powers" (COP) approximating the independent output powers associated with each source. Obviously, this is a clever use of the coherence function for a specific problem.

It is possible to compute the coherent output power directly. In point of fact, it is simplier for an instrument manufacturer to provide COP rather than coherence. The COP requires storage of the cross spectrum and a single power spectrum while the coherence requires the cross spectrum and two power spectra. Added instrument storage increases cost for the manufacturer, hence the COP is a less expensive function to compute than the coherence. From the users standpoint,

however, the coherence function offers more utility.

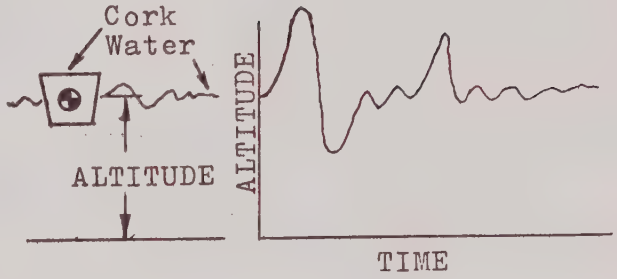
The COP (alone) cannot answer the "what's in the box?" and "where did I go wrong?" questions. In order for them to be useful intermediate steps in the solution of these questions it is necessary to have the total output power (G_{zz}) and the COP measured simultaneously. Typical instruments specifically designed for the computation of the COP do not provide for the simultaneous measurement of the total output power.

THE FOURIER TRANSFORM....

WHAT IT IS AND WHAT IT DOES

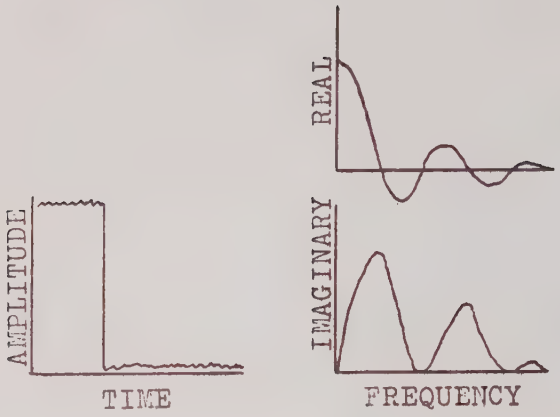
This monograph undertakes a non-rigorous introduction to the Fourier Transform, its practical implementation and application. This paper is not intended to compete with the many fine theoretical texts dealing with the Fourier Transform, rather, its intent is to create a "viseral awareness" within the reader of the far reaching capabilities of this theoretical concept to solve practical problems.

Unfortunately, existing literature contains some variations in the precise definition of the Fourier transform pair. These variations consist of multiplicative constants applied to the direct and inverse transforms. The notation used in the monograph is consistent with the currently proposed ANSI definitions for spectrum analysis and prior Nicolet Scientific (formerly Federal Scientific Corporation) monographs.



A Time History Describes an Event

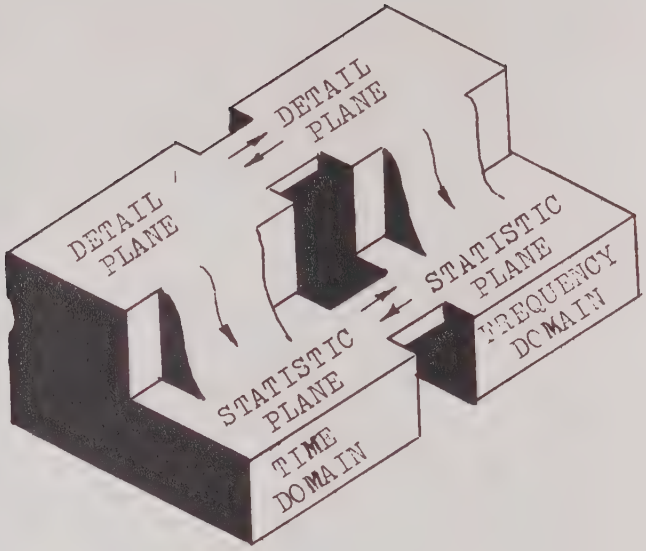
The Fourier Transform relates two alternative statements of descriptions of an event or observation. The more familiar description (of the two) is a so-called "time domain" description. In this domain, observations are understood as variation of amplitude of a quantity with time. For example, a cork floating in a body of water may undergo changes in its position (altitude) as the water undulates with time. The plotted history of these altitude excursions constitutes a time domain description of the event.



A Complex Spectrum is an Alternate Description of a Time History

The time domain description can be decomposed into a combination of sine and cosine time histories. The operation of performing this decomposition is called Direct Fourier Transformation. The act of reconstructing the time history from its harmonic constituents is called Inverse Fourier Transformation. The result of Direct Fourier Transformation is frequently called a complex spectrum. The complex spectrum (processing a real and an imaginary part) is a complex function of repetition rates or frequencies of the sine and cosine waves. That is, at any frequency the complex spectrum has a complex amplitude consisting of a real value and an imaginary value. The real value is the amplitude of the cosinusoid component at that frequency, while the imaginary part is the amplitude of the sinusoid component at the same frequency.

In a broader sense, the time domain observation could be more complex than a simple time history. The observation might consist of a computed time function such as an Auto-Correlation Function (ACF). The ACF is a time domain statistical function, describing (among other properties) the mean value, variance and possible periodicity of a time history. The ACF can be Fourier Transformed to yield a corresponding frequency domain statistical function, the Power Spectral Density (PSD). The PSD contains exactly the same information as the ACF but "rearranges" it to make the distributuion of periodic components of the signal's variance obvious. The PSD can also be computed directly from an ensemble of complex spectra. Because an ACF can be computed by Inverse Fourier Transforming a PSD, it is obvious that there are two means of computing either statistical function.



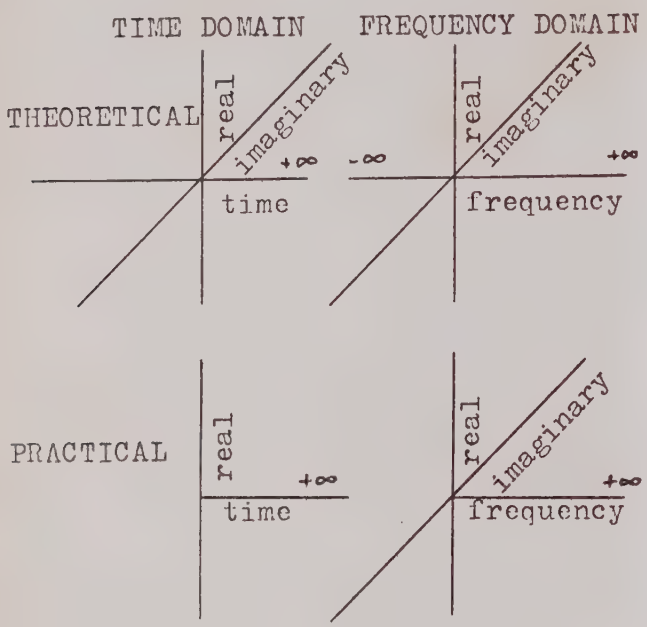
The Fourier Transform Bridges the Gap Between the Time and Frequency Domains

The Fourier Transform may be construed as a bridge relating two domains. Each domain may be visualized as having two planes of existence, namely, detail and statistic. A time history would "inhabit" the time detail plane, an ACF the time statistic plane, a complex spectrum the frequency detail plane, and a PSD the frequency statistic plane. In either domain, the act of moving from the detail plane to the statistic plane is basically irreversible as some form of averaging process is involved. That is, the statistics of a signal can be computed from the detailed description of that signal, but the detailed description cannot be reconstructed from the statistics. Sliding down from the detail plane to the statistic plane is a relatively simple business while attempting to climb back up is virtually impossible. In contrast, moving from the time domain to the frequency domain by Fourier Transformation permits crossing in both directions with equal ease. When crossing from the time domain to the frequency domain the "route" is called Direct Fourier Transformation. The return "trip" is called Inverse Fourier Transformation. The Fourier Transform bridge has two levels, one connecting the detail planes and the other connecting the statistic planes. The "structure" or implementation of the Fourier Transform is identical at either level. Understanding the nature of the bridge's structure entails some mathematics.

In a strict mathematical sense the Fourier Transform relates two complex functions of two continuous real variables (time and frequency) each of which may range between plus and minus infinity. The most common statement of this general relationship is the exponential integral pair:

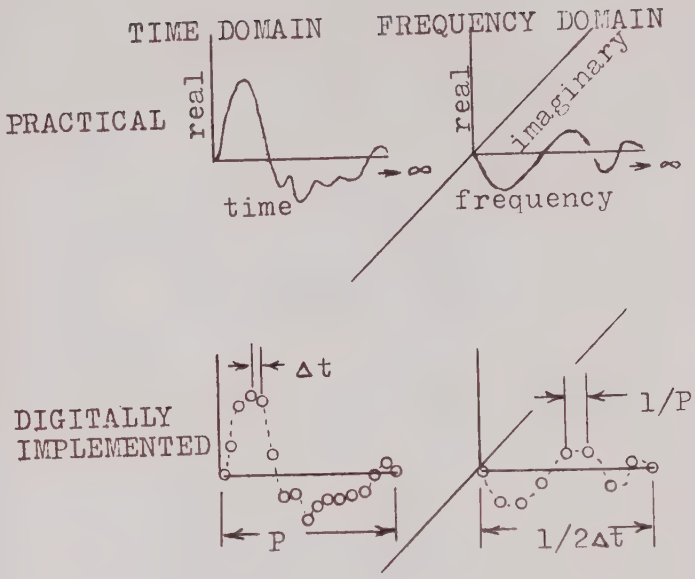
$$Ia \quad S(f) = \int_{-\infty}^{\infty} X(t) e^{-j2\pi ft}\, dt$$

$$Ib \quad X(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi ft}\, df$$



Practical Considerations Simplify the Two Domains Related by the Fourier Transform

In a practical sense the Fourier Transform is applied to time domain signals that are real rather than complex. Because time domain descriptions are real, the equivalent frequency domain descriptions are hermetian. This means that a practical spectrum need only contain positive frequency terms as the negative frequency terms contain redundant information. Further, time histories are only defined from the time point at which observation begins. This point can be defined as time zero, hence practical time histories need only contain positive time.



Synchronous Sampling in the Time Domain Band-limits the Frequency Domain. Time Limiting the Observation Period Samples the Frequency Domain.

When the Fourier Transform is implemented by digital electronics, two further restrictions are placed on the mathematical concept. Firstly, the direct transformation is applied to synchronously sampled time histories rather than continuous signals. This imposes the restriction that the resultant frequency domain spectrum have a finite upper frequency limit equal to one half of the synchronous sample rate. This restriction is necessary because the action of sampling the data restricts the range of frequencies within the data that can be identified without ambiguity. Thus sampling in the time domain band limits the frequency domain. Secondly, the direct transformation is implemented on a time domain observation of finite (rather than infinite) duration. In consequence, the resultant spectrum can only separate harmonic components that differ by an integer number of cycles within the observation time period. That is, the resultant spectrum will only contain terms at frequencies which are integer multiplies of the reciprocal of the observation time period. Thus duration-limiting in the time domain produces sampling in the frequency domain.

In practice, the theoretical transform integrals for s(f) and x(t) reduce to summations over finite limits. The resultant transform equations are called the Discrete Fourier Transform (DFT)pair, and may be stated:

$$\text{IIa} \qquad S_k = \frac{1}{N}\sum_{i=0}^{N-1} X_i e^{-j2\pi ki/N}$$

$$\text{IIb} \qquad = S_k\text{real} + j\, S_k\text{imaginary}$$

$$X_i = 2\sum_{k=0}^{N/2} S_k e^{j2\pi ki/N}$$

The Discrete Fourier Transform pair is an algebraically satisfying set of relationships. That is, if N time points are sampled, the resultant spectrum will contain exactly N unique numbers. If n is an odd number, the transformation of N real numbers will yield (N + 1)/2 complex numbers, with the imaginary part of the first (DC term) always equal to zero. If N is an even number, the transformation of N real numbers will yield (N/2) + 1 complex numbers, with the imaginary part of the first and last always equal to zero. In either case the resultant complex spectrum contains exactly N unique numbers.

The numerical evaluation of a Discrete Fourier Transform involves a succession of multiply-add operations. Certain intermediate step product-sums are redundantly calculated if the equations are directly implemented. Computational procedures have been developed that exploit this redundancy and certain symmetry of partial terms that result if the number of points, N, is selected to be a product of prime numbers. Such procedures are generically termed Fast Fourier Transform (FFT) algorithms. FFT procedures become especially efficient when N is selected as an integer power of 2, that is, an exact binary number.

It should be noted that the term, Fast Fourier Transform does not define a method of calculation. Rather, FFT denotes the fact that a Discrete Fourier Transform is being implemented with some prior planning to minimize its computation time. The actual procedures for such an implementation can run a broad gambit ranging from relatively slow "in place" algorithms available for general purpose computers to the high speed, sophisticated "Pipeline/Butterfly" processor used in the Nicolet Scientific OF-400 Series Analyzers.

The frequency domain description resulting from Fourier Transforming a time history may be expressed in several ways. The real and imaginary parts of the complex spectrum S(f) are sometimes best understood by expressing the transform pair in trigonometric form (rather than the previously discussed exponential form). This is accomplished by substituting the Euler identities into equations II, yielding:

$$\text{IIIa} \qquad S_k\text{real} = \frac{1}{N}\sum_{i=0}^{N-1} X_i \cos(2\pi ik/N)$$

$$S_k\text{imaginary} = \frac{1}{N}\sum_{i=0}^{N-1} X_i \sin(2\pi ik/N)$$

$$\text{IIIb} \qquad X_i = 2\sum_{k=0}^{N/2} (S_k\text{real}\cos(2\pi ik/N) + S_k\text{imaginary}\sin(2\pi ik/N))$$

To enhance comprehension of the direct transform consider the analysis of a signal, x(t), containing a DC offset (mean value), a sine wave and a cosine wave. For sake of simplicity assume that the sine wave is of frequency $m\beta$ and vector amplitude S while the cosine wave is of frequency $n\beta$ and amplitude C. Further assume N samples of x(t) are observed during a period of $1/\beta$. That is:

Original Signal

$$X(t) = M + S\sin(2\pi m\beta t) + C\cos(2\pi n\beta t)$$

Samples Signal

$$X_i = M + S\sin(2\pi mi) + C\cos(2\pi mi) \qquad 0 \leq i \leq N-1$$

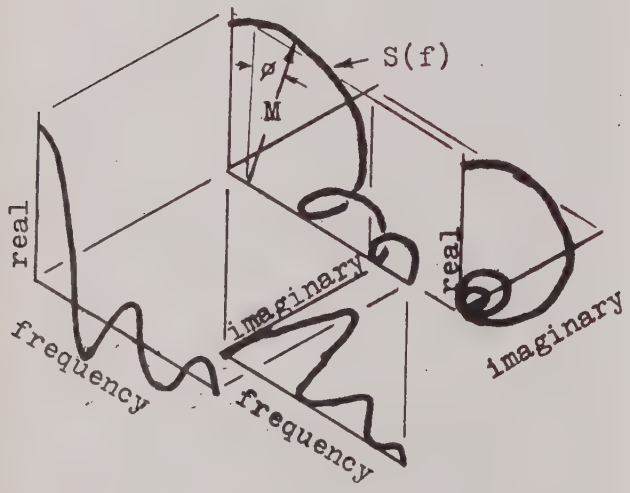
The resultant complex spectrum has non-zero values only at frequencies 0, $m\beta$ and $n\beta$. At 0 frequency (DC) the real amplitude, S_o real = M, is the mean value of the sampled function. At frequency $m\beta$ the imaginary amplitude, S_m imaginary = S/2, is one half the vector amplitude of the constituent sinusoid. Finally, at frequency $n\beta$ the real amplitude, S_h real = C/2, is one half the vector amplitude of the component cosinusoid. Hence the Direct Fourier Transform detects the presence, amplitude and relative phasing of the signal's constituents. It should be noted that the discrete direct transform is twice as sensitive to the DC term as it is to any harmonic term. Examination of equations III will reveal that the Discrete Fourier Transform is really a Fourier Series constructed over the arbitrarily selected period $1/\beta$. This is consistant with the realization that the discrete transform is an approximation of the infinite limit integral.

The complex spectrum, S(f), may be viewed in Polar Coordinates rather than the previously discussed Cartesian Coordinates. This alternate description translates the real and imaginary parts of S(f) into a magnitude and phase angle. Specifically, the magnitude is the vector resultant of the real and imaginary component functions such that:

IVa $$|S_k| = \sqrt{S_k^2\text{real} + S_k^2\text{imaginary}}$$

The phase angle is that angle measured from the real component of the complex spectrum, S_k real, to the magnitude vector, S_k. The phase angle function is calculated from the real and imaginary components of S(f), such that:

IVb $$\phi_k = \tan^{-1}\left|\frac{S_k\text{imaginary}}{S_k\text{real}}\right|$$



Alternative Views of the Complex Spectrum

In fact, a complex spectrum is a three dimensional entity, requiring real, imaginary and frequency axes for its complete presentation. The cartesian description is merely the projection of the complex spectrum on the real/frequency and imaginary/frequency planes. An alternate view is the so-called Nyquist display, a cross plot of S(f) real versus S(f) imaginary. This description is simply the projection of the complex spectrum on the real/imaginary plane. The Polar format is a curvilinear description whose magnitude is the length of a vector normal to the frequency axis and intersecting the complex function. The associated phase angle is the inclination of the magnitude vector from a line within the real/frequency plane perpendicular to the frequency axis and intersecting the magnitude vector.

It is worth noting that the magnitude distribution of a complex spectrum (viewed in polar co-ordinates) is determined solely by the shape of its corresponding time domain description. The phase distribution is determined by that shape and the position of the time function within the period of observation. Repositioning the time description within the observation period linealy twists the complex spectrum about the frequency axis. This, in effect, changes the distribution of the real and imaginary components (and therefore phase angles) but does not change the magnitude distribution. The positional independence of the magnitude distribution makes it a logical display choice when frequency content is the major interest of a measurement.
The distribution of phase angles contains the information necessary for temporal reconstruction (positioning) of a time history, while the amplitude distribution determines the relative contributions of the constituent harmonic functions.

Squaring the magnitude of a complex spectrum can be accomplished by multiplying the spectrum by its complex conjugate. That is:

V $$\left|S_k^2\right| = S_k \cdot S_k^* = (S_k\text{real} + S_k\text{imaginary}) \times (S_k\text{real} - S_k\text{imaginary})$$
$$= (S_k^2\text{real} + S_k^2\text{imaginary})$$

Note that the resultant product is real rather than complex. This result can be further understood by remembering that the product of two complex numbers (expressed in polar form) is obtained by multiplying their magnitudes and adding their phase angles. The operation of conjugation (represented by an asterisk superscript) changes the sign of the imaginary part, and thus the sign of the phase angle. Hence, the conjugate product of a complex spectrum with itself has a zero phase angle at all frequencies.

The squared spectrum $S(f)^2$, contains information about the frequency content of the original time history but none of the phase information necessary to recontruct it. In a sense, $S(f)^2$ is frequency domain statistic. The magnitude squaring process tends to expand the range of spectral amplitudes, accentuating the dominent frequencies. Ensemble averaging the squared spectrum increase the confidence of the measurement (in both a lay and statistical sense). The ensemble averaging process is defined as the point-by-point averaging of a series of (M) squared spectra such that:

VI $$\overline{\left|S_k^2\right|} = \frac{1}{M}\sum_{m=1}^{M}\left|S_k^2\right|_m$$

When dealing with broad-band signals it is frequently desirable to scale resultant spectra so that the magnitude at any (non-zero) frequency is equal to the Root Mean Square (RMS) value of the component function at that frequency. The squared spectrum then has the units of (RMS value)^2 and is generically termed a Power Spectrum (PS). That is:

$$\text{VII} \qquad PS = 2\left|\overline{S^2(f)}\right|$$

The amplitude of the Power Spectrum, PS(f) is a function of the observation period ($1/\beta$) and therefore its reciprocal (β), the resolution bandwidth. To facilitate comparison of Power Spectra measured at different resolution bandwidths a normalization of the data with respect to the resolution (β) is required. Such normalized spectra are called Power Spectral Density (PSD) functions. The PSD is defined:

$$\text{VIII} \quad PSD = G_{xx}(f) = \frac{1}{\beta}(PS(f)) = \frac{2}{\beta}\left|\overline{S^2(f)}\right|$$

Hence the PSD is obtained by appropriately scaling the ensemble average of a series of squared spectra. The PSD may also be understood as the frequency derivative of the time history's mean square value. If the signal is of zero (no DC component) the PSD is the derivative of the signal's variance. That is, integrating the PSD with respect to frequency between the frequency limits f_1 and f_2 yields that portion of the signals total variance contributed by harmonic terms of frequencies f_1 to f_2.

Inverse Fourier Transforming the PSD yields an important time domain statistical function, the Auto Correlation Function. The ACF is a measure of how similar a time history is to time delayed replicates of itself. This function is a highly sensitive detector of periodicity within a time history. The ACF is a function of a dummy time variable, τ, the amount of delay between the time history and a specific replicate. The ACF may be evaluated directly from a time history, X(t). The direct time domain computation of the ACF involves the repetitive application of a product averaging process of the form:

$$ACF = R_{xx}(\tau) = \lim_{P\to\infty} \frac{1}{P}\int_0^P X(t).X(t+\tau)\,dt$$

The ACF is symmetric about τ equal to zero and always has a maximum value at that point. In fact, the value of the ACF at τ equals zero is the mean square of x(t). Further, at values of τ approaching infinity the ACF approaches the square of the mean of x(t).

When implementing the ACF on a fixed number of synchronously sampled data points from x(t) the number of mathematical operations (and therefore the computation time) is actually reduced by first computing a PSD and then inverse transforming it. This approach does introduce additional theoretical limits but lends itself to integration in an FFT based analysis system.

The concept of correlation becomes more powerful when extended to the comparison of two signals. Such a measurement is called a Cross Correlation Function (CCF) and is defined:

$$\text{X} \quad CCF = R_{xx}(\tau) = \lim_{P\to\infty} \frac{1}{P}\int_0^P X(t).Y(t+\tau)\,dt$$

The CCF is a detector of similarity between the time histories x(t) and y(t). Unlike the ACF it is not (necessarily) symmetric about τ equal zero, nor does it (necessarily) maximize at that point. In fact, the CCF maximizes at that lag time that best aligns the two signals for maximum similarity. Thus the CCF is a useful function for determining transmission path delays. The maximum value of the CCF is bounded by the product of the RMS values of x(t) and y(t). As τ approaches infinity the CCF approaches the product of the means of x(t) and y(t).

Fourier Transforming a CCF to the frequency domain yields its counterpart, the Cross Spectral Density (CSD). The CSD can also be computed directly in the frequency domain by appropriately scaling the ensemble average of a series of conjugate products. Specifically, given the time histories x(t) and y(t) with associated complex spectra X(f) and Y(f), the CSD is defined:

$$CSD = G_{xy}(f) = \frac{2}{\beta} Y(f) \cdot X^*(f)$$
$$= G_{xy}\ \text{real} + G_{xy}\ \text{imaginary}$$

Note that CSD (like the PSD) is normalized on the resolution bandwidth, $\mathcal{B}$. Unlike the PSD, the CSD is (in general) a complex function. Each constituent of the conjugate product ensemble is formed by direct transforming observations (of $1/\mathcal{B}$ duration) from x(t) and y(t). The complex spectrum, X(f) is conjugated and multiplied with Y(f). Thus the magnitude of the product (at any frequency) is the product of the magnitude of X(f) and Y(f). The phase angle (at any frequency) is the difference between the phase associated with Y(f) and that associated with X(f). As with the PSD, the confidence of a CSD measurement is improved by ensemble averaging the conjugate product. The averaging process is denoted by an over-bar.

The CSD is a complex spectrum and may be viewed in cartesian coordinates, polar coordinates, or a Nyquist display. The CSD is a frequency domain statistic containing the same information as the time domain ACF, but "rearranged" to describe the coincidence of harmonic content in Y(f) and X(f). In fact, the CSD is the frequency derivative of the co-variance of x(t) and y(t). Simplictically, the CSD magnitude (at any frequency) indicates the presence of activity (at any frequency) in both X(f) and Y(f) while the phase angle denotes the relative phase angle between the components in X(f) and Y(f).

The cross properties (CCF and CSD) of two signals are frequently of interest when investigating the input/output relationships of a system. The PSD and CSD functions facilitate the computation of another frequency domain statistic, the Coherence Function (γ^2). The coherence function answers the important question: "Is there a linear cause/effect relationship between this system's input and output?" If the answer to this question is affirmative, the PSD and CSD functions can be further used to determine the nature of that cause/effect relationship by defining the system's Transfer Function.

Consider a system yielding an output, y(t), in response to an input excitation, x(t). By Fourier Transforming x(t) and y(t) the input and output spectra, X(f) and Y(f), respectively, are obtained. Conjugate product ensemble averaging permits computation of the input PSD, $G_{xx}(f)$, the output PSD, $G_{yy}(f)$, and the CSD, $G_{xy}(f)$. These functions allow the formation of a "squared power" ratio, the coherence function, γ^2. Specifically:

XI $$\gamma^2(f) = \frac{G_{xy}(f) \cdot G_{xy}^*(f)}{G_{xx}(f) \cdot G_{yy}(f)}$$

In the ideal situation both x(t) and y(t) could be measured without external noise contamination. Further, the system being investigated would generate output only in response to the measured input (and no other excitation source). Finally, the system's behavior would be entirely (and exactly) linear. Given such ideal conditions the coherence function equals 1, as the product of the input and output PSD's equals the square of the CSD. That is:

XII $$G_{xy}^2(f) = G_{xy}(f) \cdot G_{xy}^*(f) = G_{xx}(f) \cdot G_{yy}(f)$$

Unfortunately, the "real world" is too perverse to tolerate such simplistic behavior in anything. In a practical situation it is normally possible to obtain an uncorrupted measurement of the system's input, x(t). This is true because the excitation, x(t), is frequently defined by the observer as a part of the experiment's design. The output measurement then bears the burden of all the previously defined corruptions. This can be construed as measuring a signal, z(t), that contains the ideal output, y(t), and an unrelated error or "noise" signal, n(t). That is:

$$z(t) = y(t) + n(t)$$

The measured output PSD, $G_{zz}(f)$, contains an ensemble average of terms including the influence of both the output spectrum Y(f), and the noise spectrum, N(f). Specifically each constituent of the squared spectrum ensemble has the form:

$$\begin{aligned} Z^2(f) &= Z(f) \cdot Z^*(f) \\ &= (Y(f) + N(f)) \cdot (Y(f) + N(f))^* \\ &= Y(f) \cdot Y^*(f) + N(f) \cdot N^*(f) + Y(f) \cdot N^*(f) \\ &\quad + Y^*(f) \cdot N(f) \\ &= Y^2(f) + N^2(f) + 2YN(f) \end{aligned}$$

The term, 2YN(f), is real (only) as the complimentary cross terms have equal but opposite phase angles. Ensemble averaging reduces the 2YN(f) term to zero (asymptotically) as the noise signal, n(t), is uncorrelated with x(t) and therefore y(t). Hence the observed output PSD, $G_{zz}(f)$, contains only two terms, the desired output PSD, $G_{yy}(f)$, and the noise PSD, $G_{nn}(f)$. That is:

$$\text{XIII} \quad G_{zz}(f) \cong G_{yy}(f) + G_{nn}(f)$$

The measured CSD, $G_{xz}(f)$, contains an ensemble average of terms of the form:

$$Z(f) \cdot X^*(f) = (Y(f) + N(f)) \cdot X^*(f)$$

$$= Y(f) \cdot X^*(f) + N(f) \cdot X^*(f)$$

The ensemble averaging process reduces the $N(f) \cdot X^*(f)$ term towards zero because x(t) and n(t) are uncorrelated. Hence the measured cross spectrum $G_{xz}(f)$, approaches the desired system measurement, $G_{xy}(f)$. That is:

$$\text{IV} \quad G_{xz}(f) \cong G_{xy}(f)$$

Substituting equations XII, XIII and IV into the statement for the observed coherence function, γ_{xz}^2, yields:

$$\text{XV} \quad \gamma_{xz}^2 = \frac{G_{xz}(f) \cdot G_{xz}^*(f)}{G_{xx}(f) \cdot G_{zz}(f)} \cong \frac{G_{yy}(f)}{G_{yy}(f)+G_{nn}(f)}$$

For ideal conditions $G_{nn}(f)$ would be zero, yielding a measured coherence of unity. As $G_{nn}(f)$ increases the coherence function decreases. In the limit γ_{xz}^2 approaches zero, indicating a total lack of causual relationship between x(t) and y(t).

The coherence function is theoretically restricted to the range $0 \leq \gamma_{xz}^2 \leq 1$. In practice, observed coherence measurements in excess of 0.8 indicate the presence of a predominantly linear relationship between x(t) and y(t). In situations involving the comparison of two signals lineally related by a band-limiting process, the coherence function will fall off outside the passband of the process.

The manner in which a linear system acts upon an input, x(t), to produce a corresponding output, y(t), may be described by a time domain convolution integral. The convolution process forms y(t) from the system's impulsive response, h(t), and the input, x(t). The impulsive response is that theoretical time history that results from exciting the system with a theoretical rectangular pulse (dirac delta function) of unit area and zero width. (In practice, h(t) can be measured by alternative physically realizable techniques.) The convolution process may be stated:

$$\text{XVI} \quad y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x\, (t-\tau)\, d\tau$$

This rather complex action of a system on an input becomes far simplier when viewed in the frequency domain. In fact, the act of convolving in the time domain is equivalent to multiplying in the frequency domain. That is, if x(t) and y(t) are direct transformed the input/output relationship may be restated:

$$\text{XVII} \quad Y(f) = H(f) \cdot X(f)$$

The complex spectrum, H(f), is the Fourier Transform of the impulsive response, h(t). H(f) is a frequency domain description of the system's treatment of an input and is called a Transfer Function. The measurement of transfer functions is fundamental to the business of determining a system's behavior.

Obviously a transfer function can be computed from the ratio of input and output complex spectra. Specifically:

$$\text{XVIII} \quad H(f) = \frac{Y(f)}{X(f)} = \frac{Y(f) \cdot X^*(f)}{X(f) \cdot X^*(f)}$$

$$= H(f)\text{real} + H(f)\text{ imaginary}$$

The secondary operation of multiplication (of numerator and denominator) by $X^*(f)$ is consistant with the definition of complex division. More importantly, it suggests a means of using the powerful technique of ensemble

averaging to improve the measurement. Specifically, the numerator and denominator terms may be average separately prior to the division. This allows the restatement of equation XVIII in terms of a PSD and the CSD, such that:

$$\text{XIX} \qquad H(f) = \frac{G_{xy}(f)}{G_{xx}(f)}$$

Measuring H(f) in terms of separately averaged spectral densities offers five distinct advantages. These are:

(1) The "coherent" averaging utilized in equation XIX improves the certainty of the measurement in direct proportion to the square root of the number of constituents used in the ensemble averages. The direct averaging of H(f) as defined by equation XVIII will not improve the statistics of the measurement.

(2) The appropriate number of ensemble constituents needed for good definition of H(f) can be determined by utilizing the coherence function as a guide. The number of constituents required to achieve near-unity coherence over the band of interest is the minimum number of constituents needed to measure H(f).

(3) The coherence function and the transfer function can be computed from the same averaged data. This assures that the observed coherence accurately denotes the "state of health" of the transfer function measurement.

(4) The excitation function, x(t) may be of virtually any form. Its spectral content may vary (randomly) with time. In power limited situations such as shake tests of large structures the excitation may be swept narrow band or noise "shaped" to optimize coherence over a particular frequency range.

(5) Virtually all system descriptive functions (transfer, coherence, input PSD, output PSD, CSD, CCF, input ACF and output ACF) can be computed from 2 PSD's and 1 CSD. This allows the design of efficient analysis instrumentation optimized for the rapid and precise computation of these functions.

BASIC TIME DOMAIN STATISTICS

$$\bar{X} = \frac{1}{T}\int_0^T X(t)\,dt \qquad \text{Mean}$$

$$MS = \frac{1}{T}\int_0^T X^2(t)\,dt \qquad \text{Mean Square}$$

$$RMS = \sqrt{\frac{1}{T}\int_0^T X^2(t)\,dt} \qquad \text{Root Mean Square}$$

$$S^2 = \frac{1}{T}\int_0^T (X(t) - \bar{X})^2\,dt = MS - \bar{X} \qquad \text{Variance}$$

$$S = \sqrt{\frac{1}{T}\int_0^T (X(t) - \bar{X})^2\,dt} \qquad \text{Standard Deviation}$$

Summary of Important Fourier Duals

Function	Symbol	Function	Symbol
Time History	$x(t)$	Complex Spectrum	$X(f)$
Auto Correlation Function	$R_{xx}(\tau)$	Power Spectral Density	$G_{xx}(f)$
Cross Correlation Function	$R_{xy}(\tau)$	Cross Spectral Density	$G_{xy}(f)$
Impulsive Response	$h(t)$	Transfer Function	$H(f)$

Operation	Operation
Convolution	Multiplication
Multiplication	Convolution
Integration	Division by Frequency
Differentiation	Multiplication by Frequency
Time Inversion	Conjugation

FFT REFERENCES

TEXT	COMMENT
Advanced Engineering Mathematics C.R. Wylie, Jr. Third Edition Copyright 1966 McGraw-Hill Book Co., Inc.	Well written basic text with highly readable discussions of the fourier and LaPlace transforms, transfer functions, and convolution.
The Fourier Integral and Its Applications Athanasios Papoulis Copyright 1962 McGraw-Hill Book Co., Inc.	Theoretical text with a strong mathematical flavor. An exhaustive discussion of the fourier transform.
Random Data: Analysis and Measurement Procedures Julius C. Bendat and Allen G. Piersol Copyright 1972 John Wiley & Sons, Inc.	Latest version of a classic in the field. Strong statistical approach supported by procedural suggestions.
Digital Time Series Analysis Robert K. Otnes and Loren Enochson Copyright 1972 John Wiley & Sons, Inc.	Statistically oriented treatment of classical signal measurements. Slanted towards large computer users, this text contains many software "recipes" including FFT.
Random Vibration in Mechanical Systems Stephen H. Crandall and William D. Mark Copyright 1963 Academic Press, Inc.	Mechanical vibration discussed from a statistical viewpoint rather than the more common deterministic position.

A P P E N D I X I

(Follows)

USEFUL DEFINITIONS OF FUNCTIONS

DISPLAY SELECT	FUNCTION COMPUTED	DESCRIPTION
Time (Δ)	$A(t)$	Digital time history of input signal. (Display shows actual data processed)
Instant FFT (Δ) (INST SPEC)	$S_A = \mathcal{F}\{A(t)\}$ $= \lvert S_A\rvert\cos(\phi_A) + j\lvert S_A\rvert\sin(\phi_A)$	Complex spectrum of time history, A(t). Magnitude $\lvert S_A\rvert$ defines instantaneous spectral content. Phase angle, ϕ_A, describes *relative phasing* of periodic harmonic components. Absolute angles are only meaningful if analysis window is synchronized to periodic harmonic data.
LIN	$\lvert S_A\rvert$	
LOG	$20\ \mathrm{LOG}_{10}\ \lvert S_A\rvert$	
PHASE	ϕ_A	
REAL	$\lvert S_A\rvert\cos\phi_A$	
IMAG	$\lvert S_A\rvert\sin\phi_A$	
Aver Power Spec (Δ)	$G_{AA} = \overline{S_A \cdot S_A^*}$ $= \overline{S_A^2}$	Ensemble average of squared spectrum magnitude (formed by multiplying complex spectrum by its conjugate). A real (rather than complex) spectrum, defining *average power* at each frequency.
LIN	G_{AA}	
LOG	$10\ \mathrm{LOG}_{10}\ G_{AA}$	
RMS = $\sqrt{\ }$	$\sqrt{G_{AA}}$	
Aver Cross Spec	$G_{AB} = \overline{S_B \cdot S_A^*}$ $= \overline{\lvert S_A\rvert\lvert S_B\rvert}\cos(\overline{\phi_B - \phi_A}) + j\overline{\lvert S_A\rvert\lvert S_B\rvert}\sin(\overline{\phi_B - \phi_A})$	Ensemble averaged complex product of spectrum, S_B, with conjugated spectrum, S_A^*. Indicates those frequencies where *both* A(t) and B(t) have content. Phase angles indicate average phase *difference* between signals at each frequency.
LIN	$\lvert G_{AB}\rvert$	
LOG	$10\ \mathrm{LOG}_{10}\ \lvert G_{AB}\rvert$	
PHASE	$\overline{\phi_B - \phi_A}$	
REAL	$\lvert G_{AB}\rvert\cos(\overline{\phi_B - \phi_A})$	
IMAG	$\lvert G_{AB}\rvert\sin(\overline{\phi_B - \phi_A})$	
Transfer Funct	$H_{AB} = \dfrac{G_{AB}}{G_{AA}}$ $= \dfrac{\lvert G_{AB}\rvert}{\lvert G_{AA}\rvert}\cos(\overline{\phi_B - \phi_A}) + j\dfrac{\lvert G_{AB}\rvert}{\lvert G_{AA}\rvert}\sin(\overline{\phi_B - \phi_A})$	Complex ratio of cross spectrum, G_{AB}, to input power spectrum, G_{AA}, defines the gain and phase lag introduced by a transmission system *excited* by A(t) and *responding* with B(t).
LIN	$\lvert H_{AB}\rvert$	
LOG	$20\ \mathrm{LOG}_{10}\ \lvert H_{AB}\rvert$	
PHASE	$\overline{\phi_B - \phi_A}$	
REAL	$\lvert H_{AB}\rvert\cos(\overline{\phi_B - \phi_A})$	
IMAG	$\lvert H_{AB}\rvert\sin(\overline{\phi_B - \phi_A})$	

DEFINITIONS OF FUNCTIONS

DISPLAY SELECT	FUNCTION COMPUTED	DESCRIPTION
Coherent Output Power LOG	$COP = \frac{G_{AB} \cdot G_{AB}^*}{G_{AA}}$ $= \gamma^2_{AB} G_{BB}$ $10 \log_{10} COP$	Coherent output power (COP) spectrum indicates power in signal, B(t), that is coherent with signal, A(t). Total power spectrum G_{BB}, indicates entire content of B(t) while COP indicates only that portion caused by A(t) thru a linear process. When B(t) is caused by unrelated multiple inputs, COP indicates only that portion due to single input, A(t).
Coherence	$\gamma^2_{AB} = \frac{G_{AB} \cdot G_{AB}^*}{G_{AA} \cdot G_{BB}}$ $= \frac{\lvert G_{AB} \rvert^2}{G_{AA} \cdot G_{BB}}$	Real Spectrum "power squared" ratio indicates *linear cause/effect* relationship between A(t) and B(t). Linear transmission system excited *only* by A(t) yields $\gamma_{AB}^2 = 1$ at all frequencies. Non-linearities or inputs in addition to A(t) produce γ_{AB}^2 between "0" and "1".
Auto-Correlation Function (Δ)	$R_{AA}(\tau) = \mathcal{F}^{-1}\{G_{AA}\}$ $= \frac{1}{P}\int_0^P A(t) \cdot A(t+\tau)dt$	Time domain statistic comparing *similarity* of A(t) to replicates of *itself*, delayed by time, τ. At $\tau = 0$, R_{AA} = mean square of A(t). As $\tau \to \infty$, R_{AA} approaches square of mean value. Oscillations in R_{AA} indicate *bandwidth* and *periodicity* of A(t).
Cross-Correlation Function	$R_{AB}(\tau) = \mathcal{F}^{-1}\{G_{AB}\}$ $= \frac{1}{P}\int_0^P A(t)B(t+\tau)dt$	Time statistic comparing *similarity* of A(t) and B(t). Value of τ that maximizes R_{AB} indicates *time lag* between signals. Maximum value of R_{AB} divided by square root of R_{AA} and R_{BB} yields correlation coefficient (equal to 1 for identical signals, 0 for unrelated signals).
Signal Enhancement (Δ)	$\overline{A}(t)$	Ensemble averaged time histories computed at each time point across a series of instantaneous time histories, each triggered from a common condition. Used to extract mean value time history.
Probability Density Function (PDF)	$q(x) = \frac{d[Q(x)]}{dx}$ such that $\int_a^b q(x)dx = P\{a < A \leq b\}$	Statistic defining most probable amplitude of A(t).
Cumulative Distribution Function (CDF)	$Q(x) = P\{A \leq x\}$	Statistic defining probability that time signal A(t) has instantaneous amplitude of x or less.

Definition of Symbols

A(t) – Input time function on Channel A

B(t) – Input time function on Channel B

$|S|$ = Magnitude of instant FFT = $\sqrt{S^2_{REAL} + S^2_{IMAG}}$

ϕ = Phase angle of instant FFT = $\tan^{-1}\frac{S_{IMAG}}{S_{REAL}}$

j = $\sqrt{-1}$

S^* = Complex conjugate of $S = S_{REAL} - j\,S_{IMAG}$

(overbar) = Ensemble average

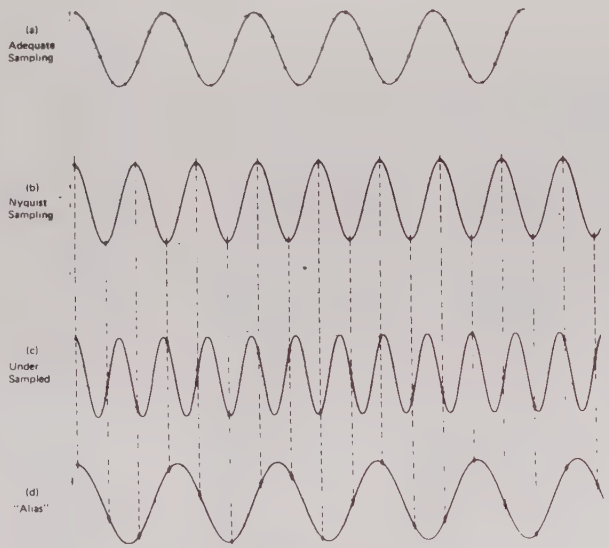
t = Time

τ = Artificially introduced time delay

$\mathcal{F}\{\ \}$ Direct Fourier Transformation

$\mathcal{F}^{-1}\{\ \}$ Inverse Fourier Transform

(Δ) Similar for Channel B



ALIASING OF SAMPLED DATA

The sampling frequency at which analog-to-digital conversion takes place in any digital analyzer, such as an FFT unit, must be compatible with the spectrum composition of the signal which is to be analyzed. In accordance with Nyquist's sampling theorem, the sampling frequency must be *at least* twice the highest signal frequency of interest.

Usually, input signals include frequency components beyond the analysis range. These components must be attenuated by means of a low-pass filter, which is called the "anti-aliasing" filter. Without proper anti-aliasing filtering, *any* digital spectrum analyzer will produce spurious frequencies. Practical anti-aliasing filters do *not* cut off (attenuate) sharply at a single frequency. Instead, the attenuation characteristic "falls off" with some finite slope.

Two frequencies are aliases of one another if sinusoids for those frequencies cannot be distinguished by the sampled values. For example, if the low-frequency sine wave shown in Fig. (a) is sampled at the rate shown by the dots, there is more than enough information to define this sine wave. At higher sine wave frequencies, the number of sample points per sine wave cycle decreases until there are exactly two samples per cycle of sine wave. At this point, the sine wave is being sampled at the so-called Nyquist rate, as illustrated in Fig. (b).

If the sine wave frequency is increased further, sampling becomes insufficient. Figure (c) shows the case of an "under-sampled" sine wave. Unfortunately, the samples taken from the sine wave in Fig. (c) *"appear"* to the digital system as if they had been derived from a lower-frequency input, as shown in Fig. (d). Observe that sample amplitudes are identical in Figs. (c) and (d).

In other words, a high-frequency input to the system will generate a false low-frequency output. Once a signal has been sampled, there is no way to resolve the ambiguity between a "valid" low-frequency signal and a spurious response due to aliasing. The only way to eliminate aliasing is to filter the input data prior to analog-to-digital conversion, thereby preventing high-frequency signals (which can cause spurious low-frequency responses) from being sampled and digitized.

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specifications

FUNCTIONS CALCULATED AND DISPLAYED (19)

1. Input time history (dual)
2. Ensemble averaged input time history (SIG ENH) (dual)
3. Instant Spectrum of input time or averaged input time
   (a) 400-line (dual) – INST SPEC, 1K CH A-B
   (b) 800-line (single) – INST SPEC, 2K CH A
4. Power spectral density - PSD (dual) – POWER SPEC, PER Hz
5. Averaged power spectrum (dual) – POWER SPEC
6. RMS spectrum (square root of averaged power spectrum (dual) – RMS SPEC
7. Cross-spectrum averaged (complex-amplitude/phase or real/imaginary) – CROSS SPEC
8. Transfer function (complex-amplitude/phase or real/imaginary) – TF
9. Equalized transfer function
10. Transmissibility (ratio of RMS spectrum B to RMS spectrum A) – TM
11. Equalized transmissibility
12. Coherence (COH)
13. Coherent Output Power (COP)
14. Impulse response – IMPUL RESP
15. Equalized impulse response
16. Probability Density Function (PD) – PROB DEN
17. Cumulative Distribution Function (CD) – CUM DIST
18. Auto-correlation – AUTO CORR
19. Cross-correlation – CROSS CORR

STORAGE OF FUNCTIONS

STORE - Any dual display (dual-channel or cross-function, such as TF, real and imaginary or magnitude and phase) can be stored together with annotation; a single time function or one 800-pt. spectrum can be stored with annotation.

RECALL - Place stored data on display screen (LED on); revert back to current data display upon second actuation of pushbutton (LED off).

COMPare - One set of stored data and current data displayed simultaneously (CH A and CH A' , TF magnitude stored and TF magnitude current, etc.).

EQL TF/TM - Equalizes transfer function if TF is stored (real and imaginary or magnitude and phase) by dividing magnitude of current by stored and subtracting phase of stored from current; also compensates impulse response and TM.

INPUT CHARACTERISTICS (Each Input Channel)

Maximum Amplitude (Volts 0 - Peak): 0.1, 0.2, 0.5, 1, 2, 5, 10, 20

Coupling: AC or DC, normal or inverted, ZERO grounds input; AC -3 dB at 0.5 Hz

Impedance: 100K ohms

Test Signal: Single peak approximately full scale appears in each channel at a frequency which is 64 percent of analysis range (normal input is disconnected when test signal is on).

ANALYSIS CHARACTERISTICS

Frequency (Time Ranges): 16 ranges from 0 - 1 Hz to 0 - 100 kHz in 1-2-5 steps (400 sec to 4 ms for 2-ch, 800 sec to 8 ms for 1-ch)

Resolution: Samples in time functions - 1024 for each of 2-ch, 2048 for 1-ch. Resolution elements in frequency functions - 400 for each of 2-ch, 800 for 1-ch.

Input Sampling: 12-bit A/D conversion at 2.56 times maximum frequency in the range selected or 256,000 per second maximum.

Input Filters: Range switch automatically selects a pair of matched, low-pass, anti-aliasing filters, which provide at least 70 dB attenuation of those input frequencies which can cause spurious responses due to aliasing (10 Hz filter used on 1, 2 and 5 Hz ranges), frequency response ±0.5 dB (except ±1.0 dB on 100 kHz range)

Transient Capture Control: Digital data in input buffer memories may be held either manually by placing the HOLD/REL button in HOLD (light on), or automatically by triggering on the transient waveform in either A or B input channels, at a preset trigger level (±1/2, ±1/4, ±1/8, or ±1/16 FS) after ARM is pushed (light on); delay or anticipation of the trigger may be set using the keyboard (T → A) in full or fractional time windows as determined by the frequency range setting.

Input Amplitude Monitors: Lights indicate peak signal levels - OVERLOAD, -6 dB (greater than 1/2 full scale), -12 dB (between 1/2 and 1/4 FS), -18 dB (between 1/4 and 1/8 FS), -24 dB (between 1/8 and 1/16 FS); electronic graticule top and bottom in time display indicates amplitude limit.

BUILT-IN ANNOTATED DISPLAY

Vertical Display Gain: In linear, full scale can be set to 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6 or 8 X 10^K; in log, from +50 dB to -50 dB in 10 dB steps; display range set from 20 dB to 80 dB in 10 dB steps via keyboard (DSPL dB).

Time Displays: Self-aligning electronic graticule, input maximum amplitude written on CRT in volts 0 - peak.

Frequency Displays: Self-aligning electronic graticule, numbers indicate input maximum amplitude of each channel, full scale frequency, averaging mode and number of spectra averaged, type of function, format, weighting, cursor amplitude in calibration units, cursor frequency, and latest keyboard-entered data.

DISPLAY FORMATS (where applicable to function selected)

Time: Ch A, ch B, ch A and ch B, correlation, impulse response

Frequency: Ch A, ch B, ch A and ch B.

Cross-Frequency: Real, imaginary, real and imaginary, magnitude, phase, magnitude and phase, Nyquist (real vs imaginary).

Probability: PD ch A, PD ch B, PD ch A and ch B, CD ch A, CD ch B, CD ch A and ch B.

CALIBRATION AT CURSOR

Marker: Single line cursor intensified each waveform at its intersection to indicate location of measurement; in HARMONIC, all multiples of cursor frequency are illuminated to indicate frequencies harmonically related to the cursor position.

Amplitude: In engineering units, using keyboard to establish EU for a reference signal or sensitivity as V/EU; dB reference can be set for relative measurements; readings can be in volts, power (V^2), or dB; readings can be normalized per Hz (such as PSD).

Frequency: In Hz (cycles/sec), CPM (cycles/minute), or orders (enter FS order via keyboard); reads absolute or as difference from reference frequency (FREQ REF set with keyboard).

Time: Seconds or millisecs; %FS in "orders" (external sampling).

CONTROL STORAGE

Stores and recalls nine sets of control settings. Each set includes a selection in each of the following control groups:

CURSOR TYPE	DELAY T→A	DISPLAY FORMAT
CURSOR READ	DELAY A→B	DISPLAY MEMORY
DISPLAY GAIN	MODE	COHERENCE BLANK
AVERAGE MODE	FUNCTION	PLOT RATE
AVERAGE N	DISPLAY CH	DISPLAY dB

KEYBOARD SETTINGS

The following parameters can be set and recalled:

1. **AVG N** - Number of functions averaged
2. **DELAY T→A** - Delay (or anticipation) between trigger signal and start of Channel A data window, in time windows or fraction of windows.
3. **DELAY A→B** - Delay introduced between Channels A and B in time windows.
4. **PANEL** - Stores nine sets of panel settings with identification number for future recall.
5. **RMS 1→2** - Calculates total RMS power between cursor location number 1 and number 2.
6. **∫dt** - Synthetic time-domain integration or double integration after processing affects all averaged functions involving the integrated input channel (e.g. converts acceleration to velocity to displacement).
7. **d/dt** - Synthetic time-domain differentiation or double differentiation after processing affects all functions involving the differentiated input channel (e.g. converts displacement to velocity to acceleration).
8. **COH BLANK** - Sets level of display blanking for all spectra based on degree of coherence.
9. **PLOT RATE** - Sets (by code) from very slow to very fast in five steps
10. **DSPL dB** - Sets log scale from 20 to 80 dB full range in 10-dB steps.
11. **LIN (REF)** - The amplitude at cursor is equated to a numerical reference.
12. **dB (REF)** - The amplitude at cursor is set to a dB reference.
13. **V/EU** - Sets calibration sensitivity of amplitude measurements.
14. **FREQ REF** Sets cursor location as reference for frequency readings.
15. **ORDER REF** - Sets cursor location as particular order number.

AVERAGING

Type:

SUM adds N power spectra

DIFF subtracts N power spectra from pervious average

EX follows the changing statistics of a signal exponentially with a time constant of N X time window

PEAK produces the profile of the maximum amplitudes at each frequency element during N spectra

SWEEP adds power spectra with maximum redundancy until averaging is manually stopped

TRANS adds power spectra of N transients automatically, *except* those which overloaded input; after each capture the transient trigger is automatically armed.

Control:

START (erases memory and starts averaging)

STOP (halts averaging)

CONT (resumes averaging until N is reached, *or* restarts averaging to N without erasing memory)

Number set using keyboard (AVG N)

DIGITAL INTERFACES (standard built-in):

(1) IEEE Standard 488-1975 Instrumentation Interface, full listener/talker

(2) RS232 Type D Serial Interface, 660 acts as a terminal for transmission via a modem

(3) High speed parallel input/output bus with control of bus via IEEE488 or RS232

AVAILABLE OPTIONS

Digital Recording (Model 134): Produces hardcopy of total annotated display.

Frequency Expansion (-2): Built-in dual channel windows 50, 100, 500, 1000, or 2000 Hz wide, which can be located in 10-Hz steps to 100 kHz (designated "-2 (window), e.g. -2 (100) for 100 Hz window.

Mass Storage (Model 160A) Digital cartridge recorder stores functions and annotation displayed with ID numbers for search and playback.

Modal Analysis (MA): Records view of test object and displays dynamic vibrations in each mode.

MISCELLANEOUS

Weight: 36.4 kgm (80 pounds) nominal

Size: 35.56 cm (14 inches) high, 43.18 cm (17 inches) wide, 52.79 cm (20-7/8 inches) deep

NICOLET SCIENTIFIC CORPORATION

NICOLET SCIENTIFIC CORPORATION
245 Livingston Street
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Telex: 851311135

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Te. (416) 625-3901 Telex: 06-960126

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NICOLET JAPAN CORPORATION
Daisho Building (10th Floor)
8-7, 6 Chome, Nishinakajima-Cho
Yodogawa-Ku, Osaka City, Japan
Te. 06-305-2150

Engineering improvements may require specification changes

WARRANTY

Nicolet Scientific Corporation warrants each instrument manufactured by it to be free from defects in material and workmanship. Our liability under this warranty is limited to servicing or adjusting any instrument returned to the factory for that purpose and to replace any defective parts thereof. This warranty is effective for one year after delivery to the original purchaser when the instrument is returned, transportation charges prepaid by the original purchaser, and when upon examination it is disclosed to our satisfaction to be defective. If the fault has been caused by misuse or abnormal conditions of operation, normal service charges will prevail. In this case, an estimate will be submitted before the work is started. Warranty returns must first be authorized by Nicolet Scientific Corporation.

All equipment not manufactured by Nicolet Scientific Corporation is subject only to the warranty provided by the supplying manufacturer.

Nicolet Scientific Corporation reserves the right to make changes in the design of its instruments without incurring any obligation to make the same changes on equipment previously purchased.

CLAIM FOR DAMAGE IN SHIPMENT

The instrument should be tested as soon as it is received. If the instrument is damaged in any way, a claim should be filed with the carrier. A full report of the damage should be obtained by the claim agent, and this report should be forwarded to us. We will then advise you of the disposition to be made of the equipment and arrange for repair or replacement. Include model number, serial number, and purchase order number when referring to this instrument for any reason.

SHIPPING

If the unit fails to operate or any fault develops, the following steps should be taken:

1. Notify us, giving full details of the difficulty,

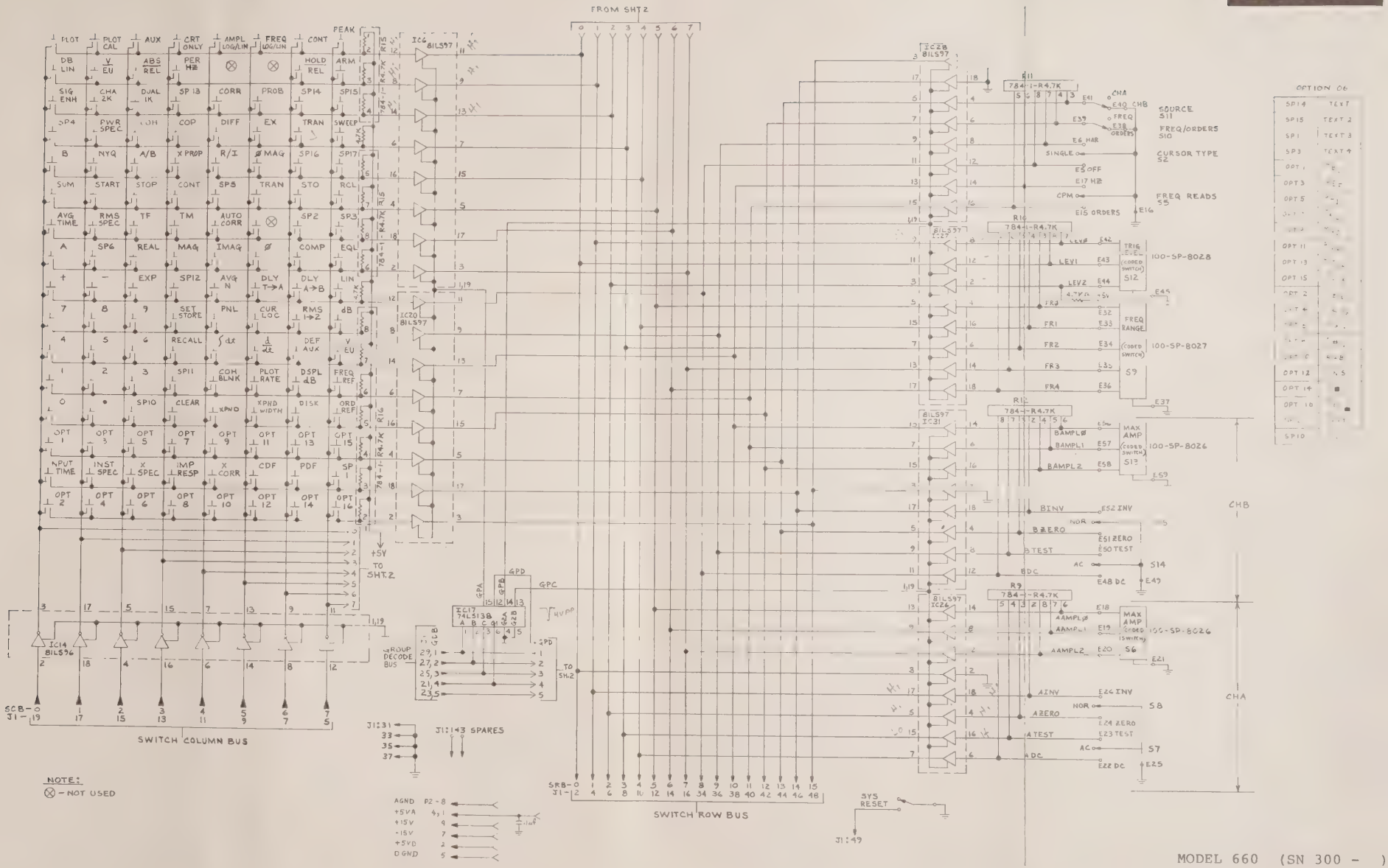
and include the model number and serial number of the equipment. On receipt of this information, we will give you service data or shipping instructions.

2. On receipt of shipping instructions, forward the instrument prepaid to Nicolet Scientific Corporation. If requested, an estimate of the charges will be made before the work begins, provided the instrument is not covered by the warranty.

3. If the instrument is to be shipped, it is imperative that it be properly packaged. Where possible, use the specially designed packaging material in which the unit was delivered.

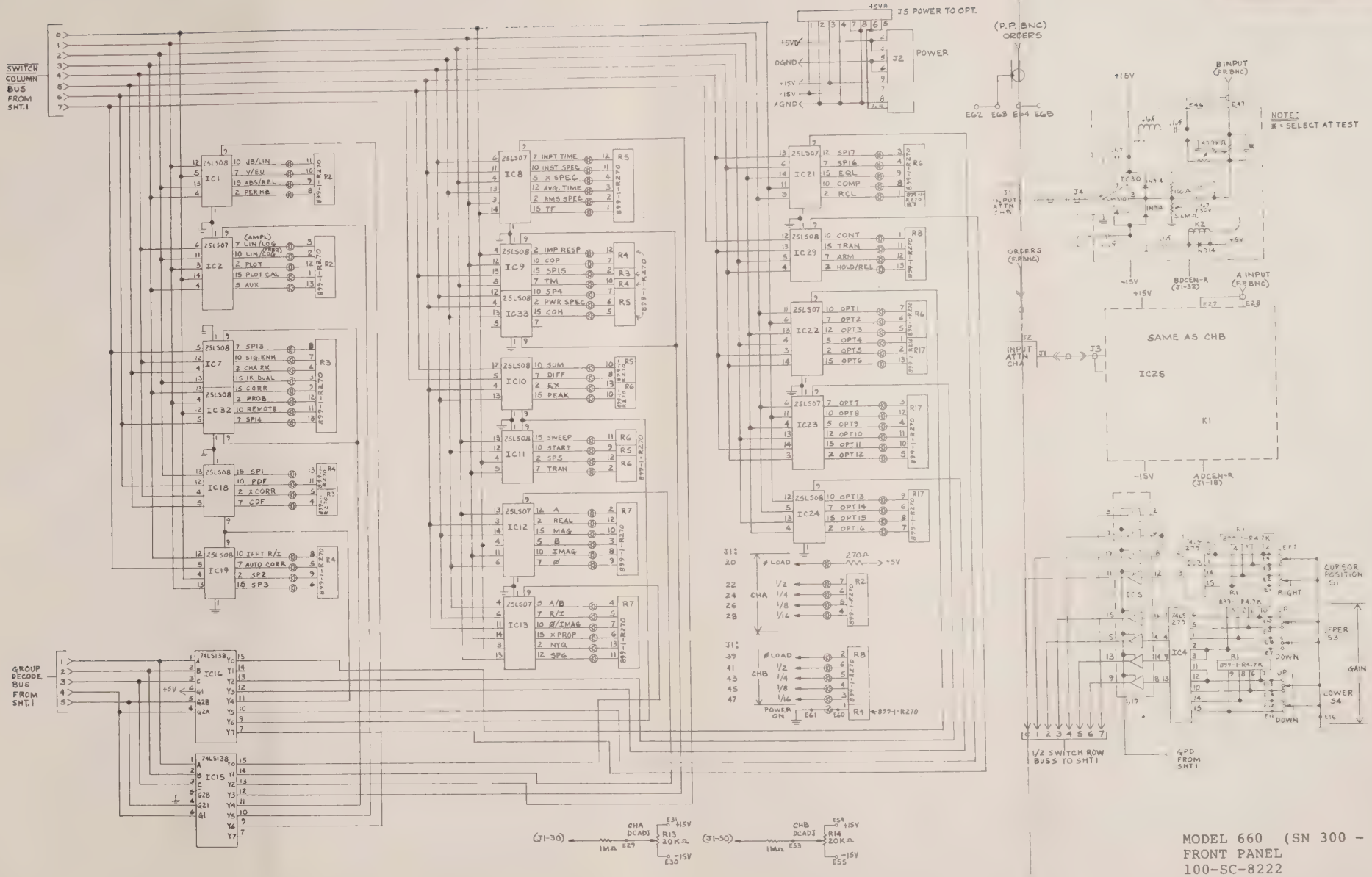
This warranty is expressly in lieu of all other obligations or liabilities on the part of Nicolet Scientific Corporation, which neither assumes nor authorizes any person to assume for it any other liability in connection with the sale of its instruments.

nicolet scientific

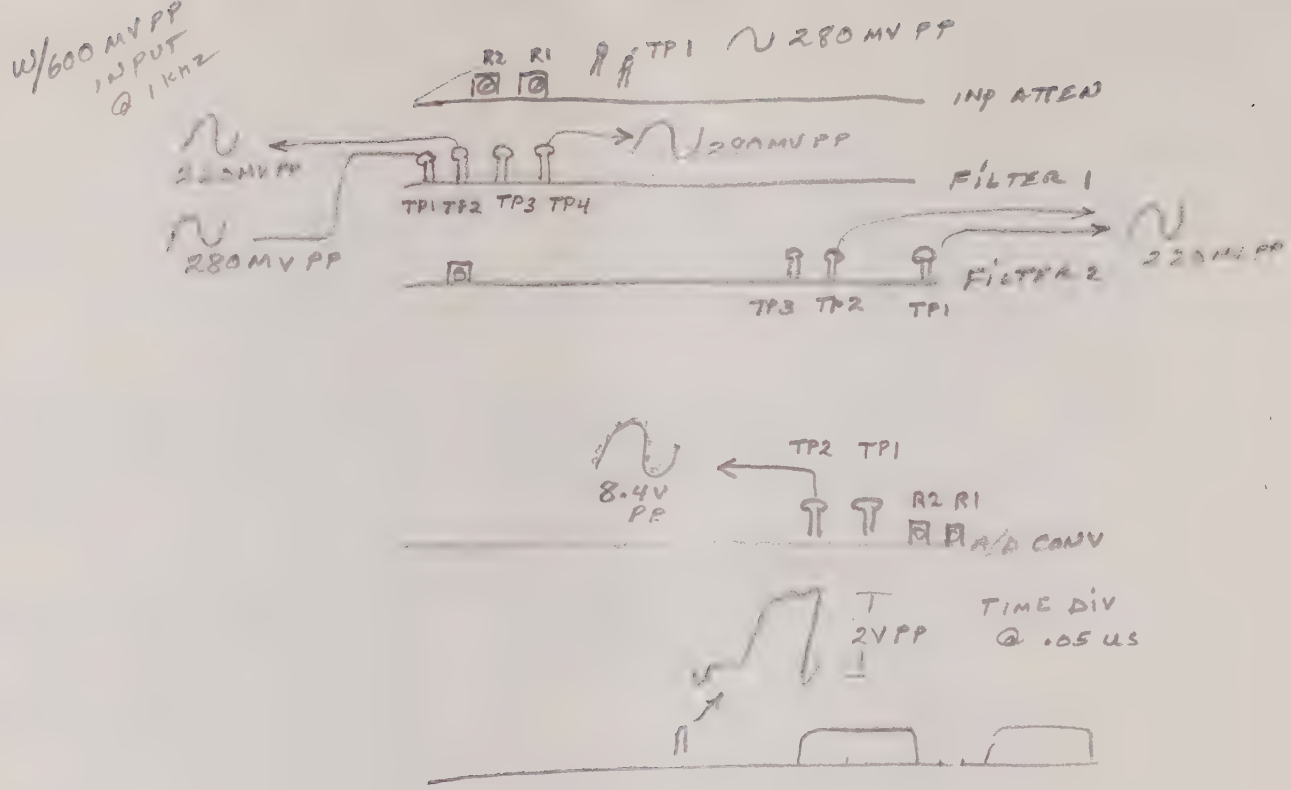


MODEL 660 (SN 300 -)
FRONT PANEL
100-SC-8222
SHEET 1 of 2 REV E
2-80

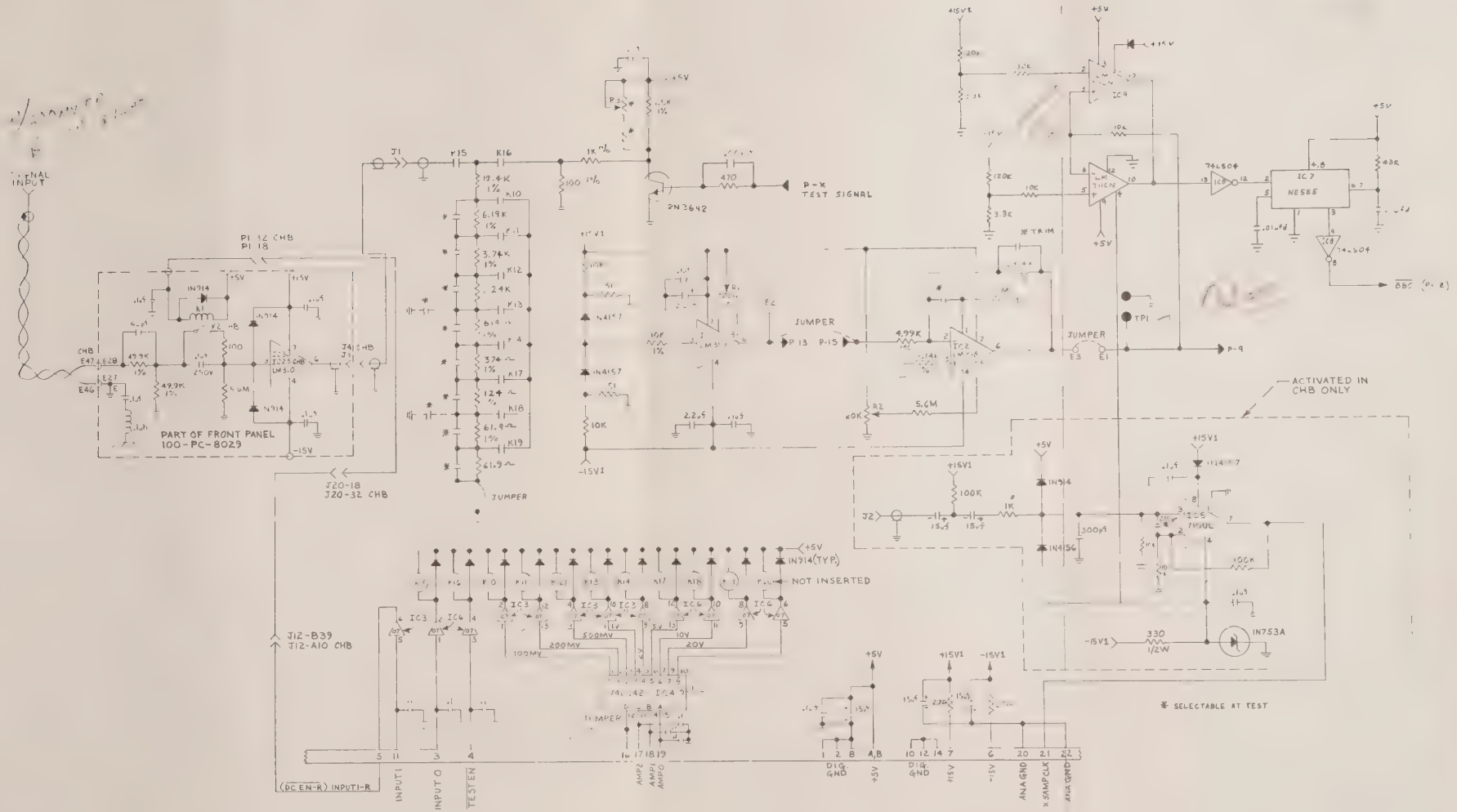
nicolet scientific



MODEL 660 (SN 300 -)
FRONT PANEL
100-SC-8222
SHEET 2 of 2 REV E
2-80

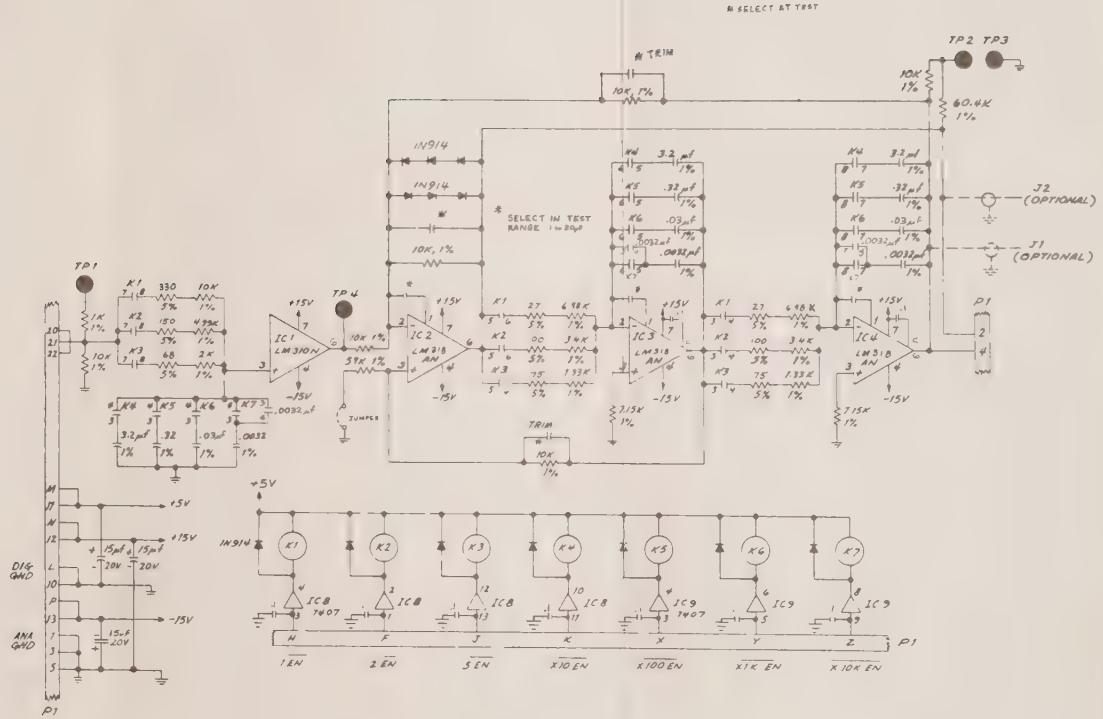

W/600 MV PP INPUT @ 1 KHz
R2 R1
TP1
280 MV PP
INP ATTEN
220 MV PP
220 MV PP
FILTER 1
TP1 TP2 TP3 TP4
280 MV PP
FILTER 2
220 MV PP
TP3 TP2 TP1
8.4V PP
TP2 TP1
R2 R1
A/D CONV
2V PP
TIME DIV @ .05 us

nicolet scientific



MODEL 660 (SN 300 -)
INPUT ATTENUATOR
100-SC-8064
SHEET 1 of 1 REV E
2-80

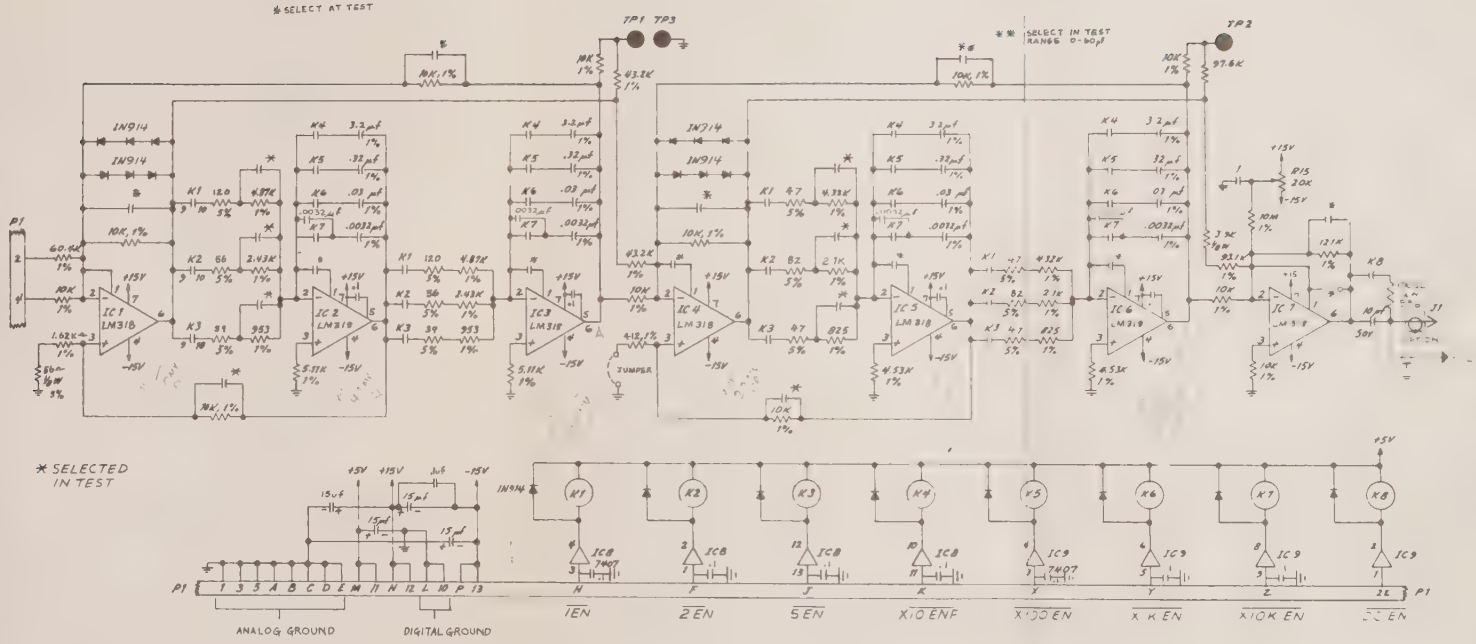
nicolet scientific



MODEL 660 (SN 300 -)
FILTER NUMBER 1
100-SC-8598
SHEET 1 of 1

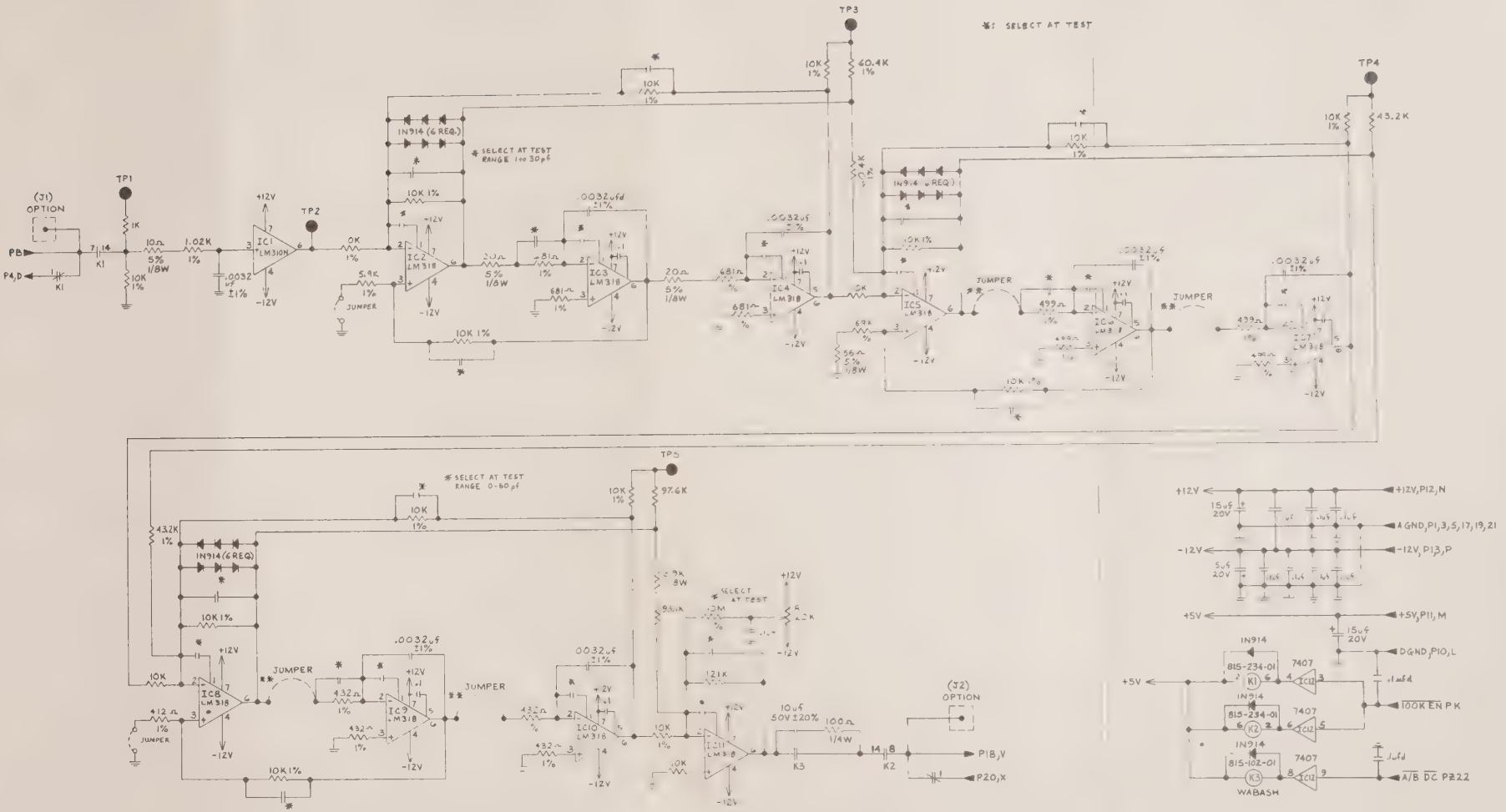
REV E
2-80

nicolet scientific



MODEL 660 (SN 300 -)
FILTER NUMBER 2
100-SC-8614
SHEET 1 of 1 REV E
2-80

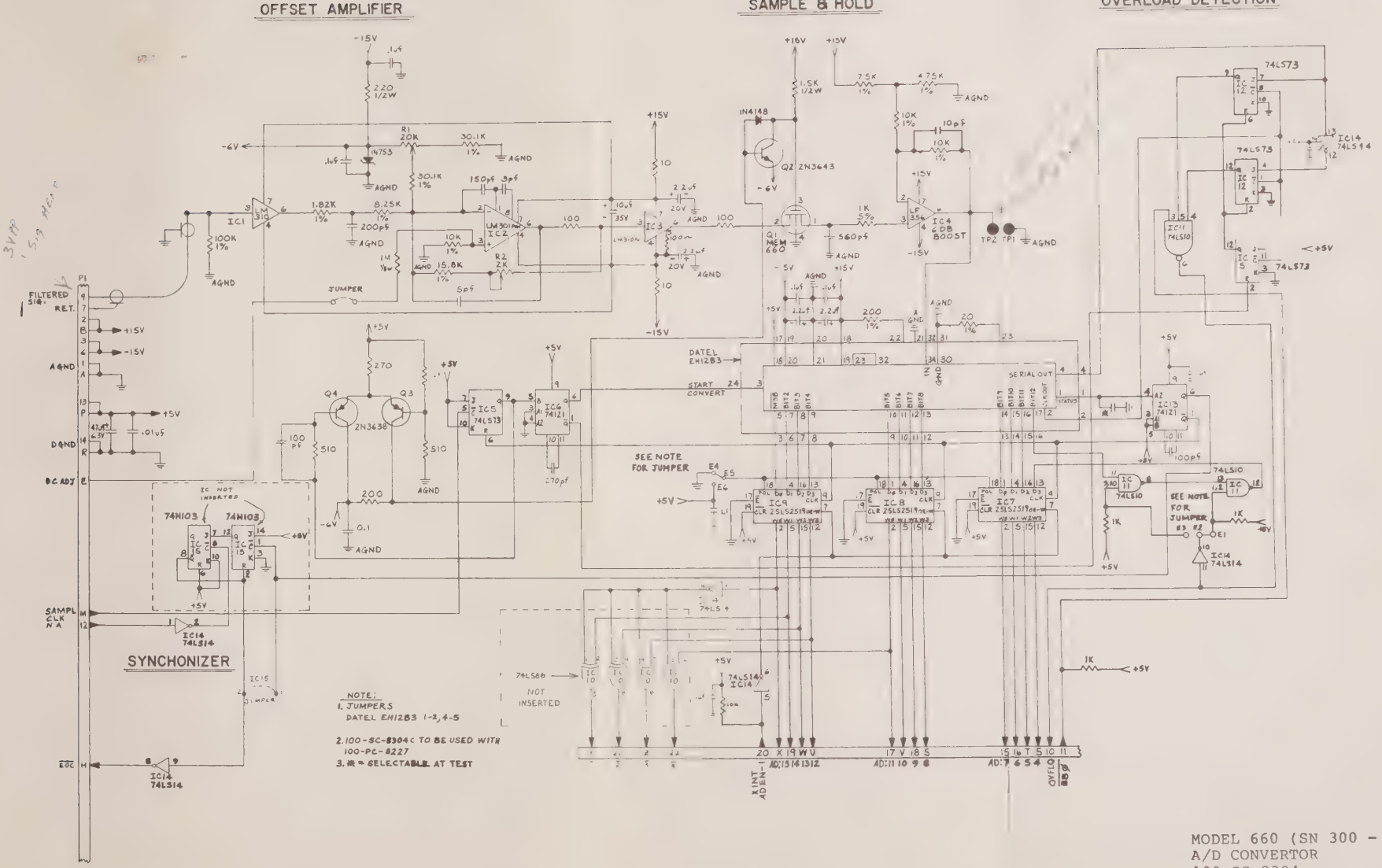
nicolet scientific



MODEL 660 (SN 300 -)
FILTER NUMBER 3
100-SC-8056
SHEET 1 of 1

REV E
2-80

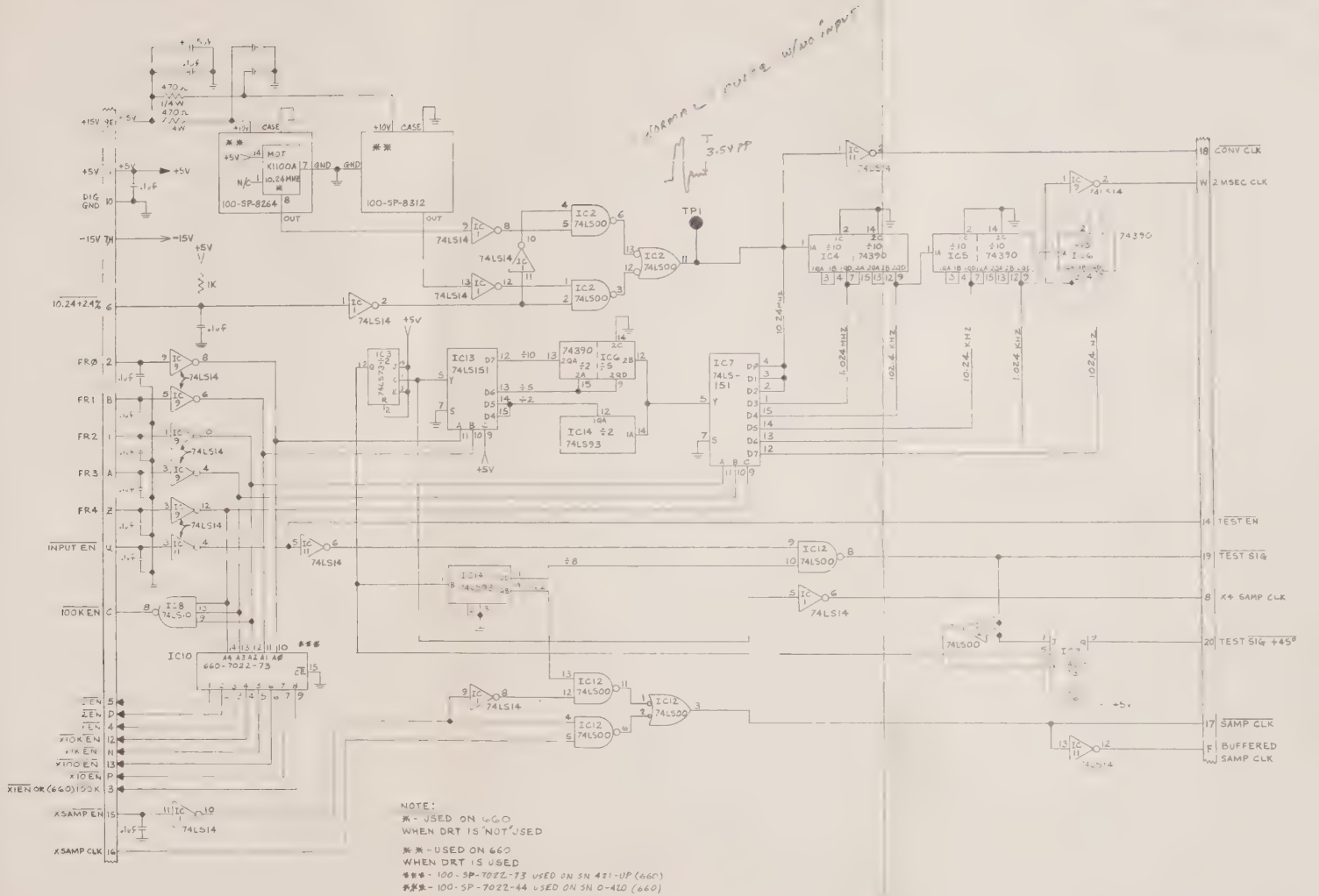
nicolet scientific



MODEL 660 (SN 300 -)
A/D CONVERTOR
100-SC-8304
SHEET 1 of 1 REV E
2-80

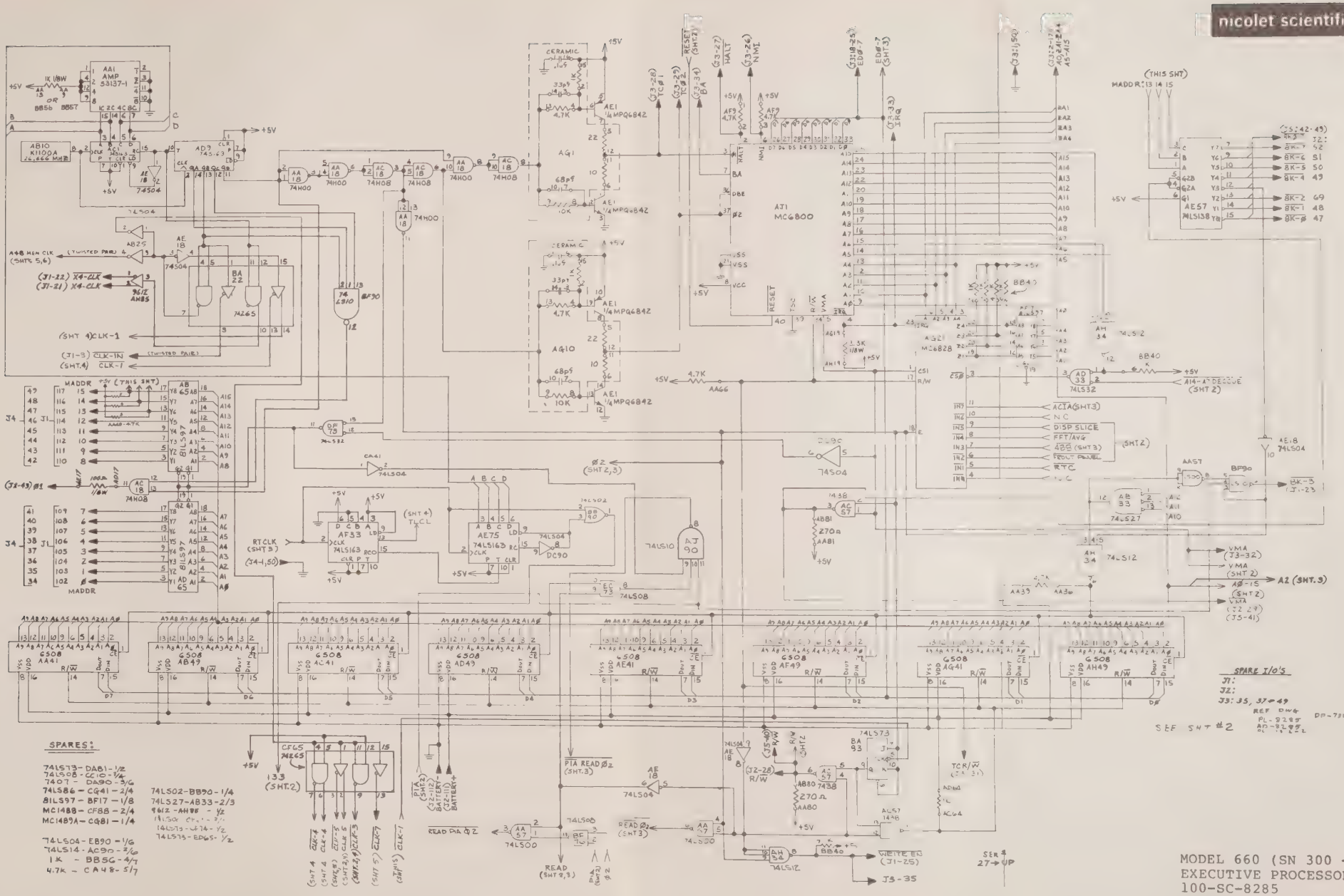
nicolet scientific

OPT 2D HAS 2 OSC.
2 GREEN CANS

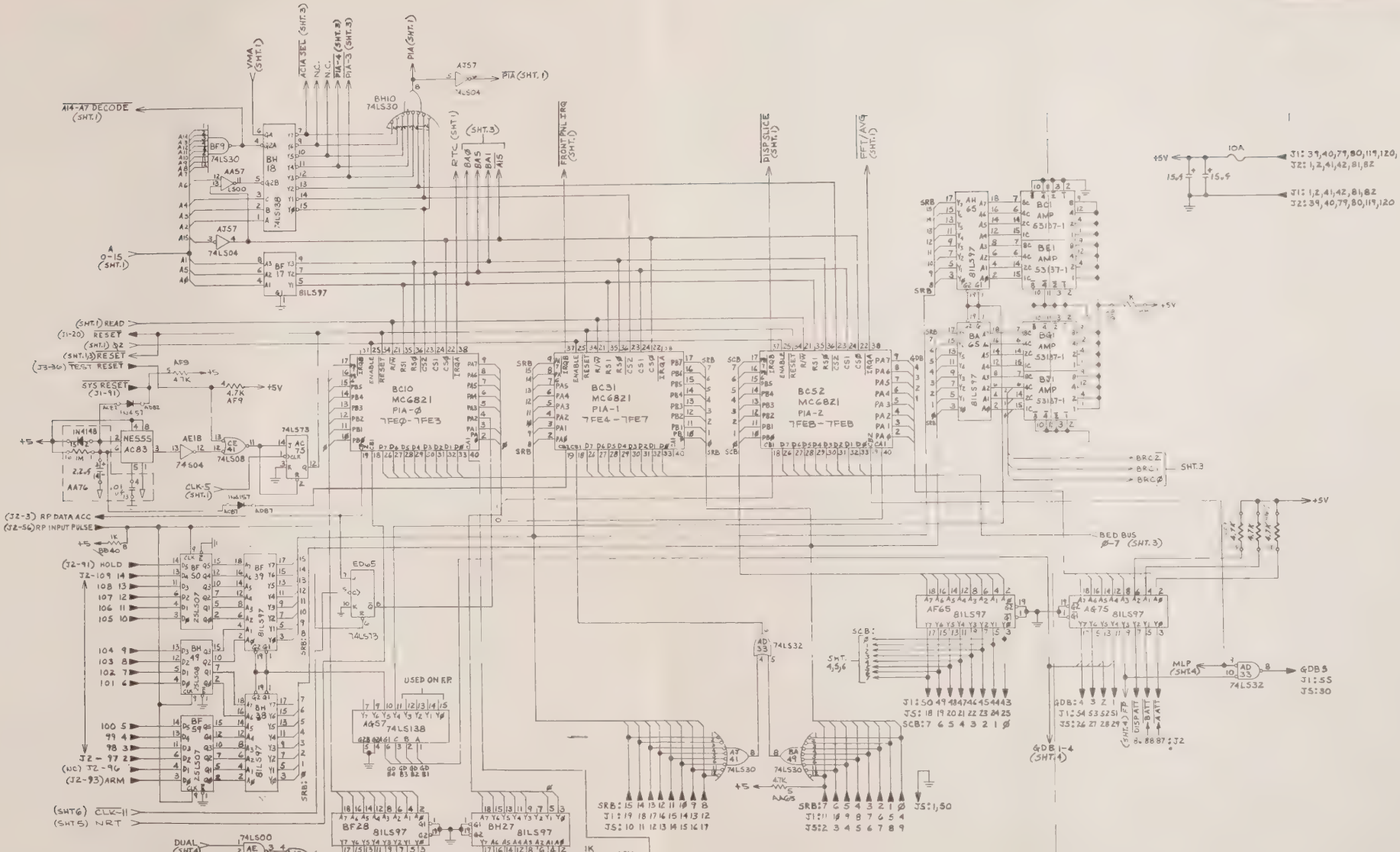


MODEL 660 (SN 300 -)
SAMPLING CONTROL
100-SC-8313
SHEET 1 of 1 REV E
2-80

nicolet scientific

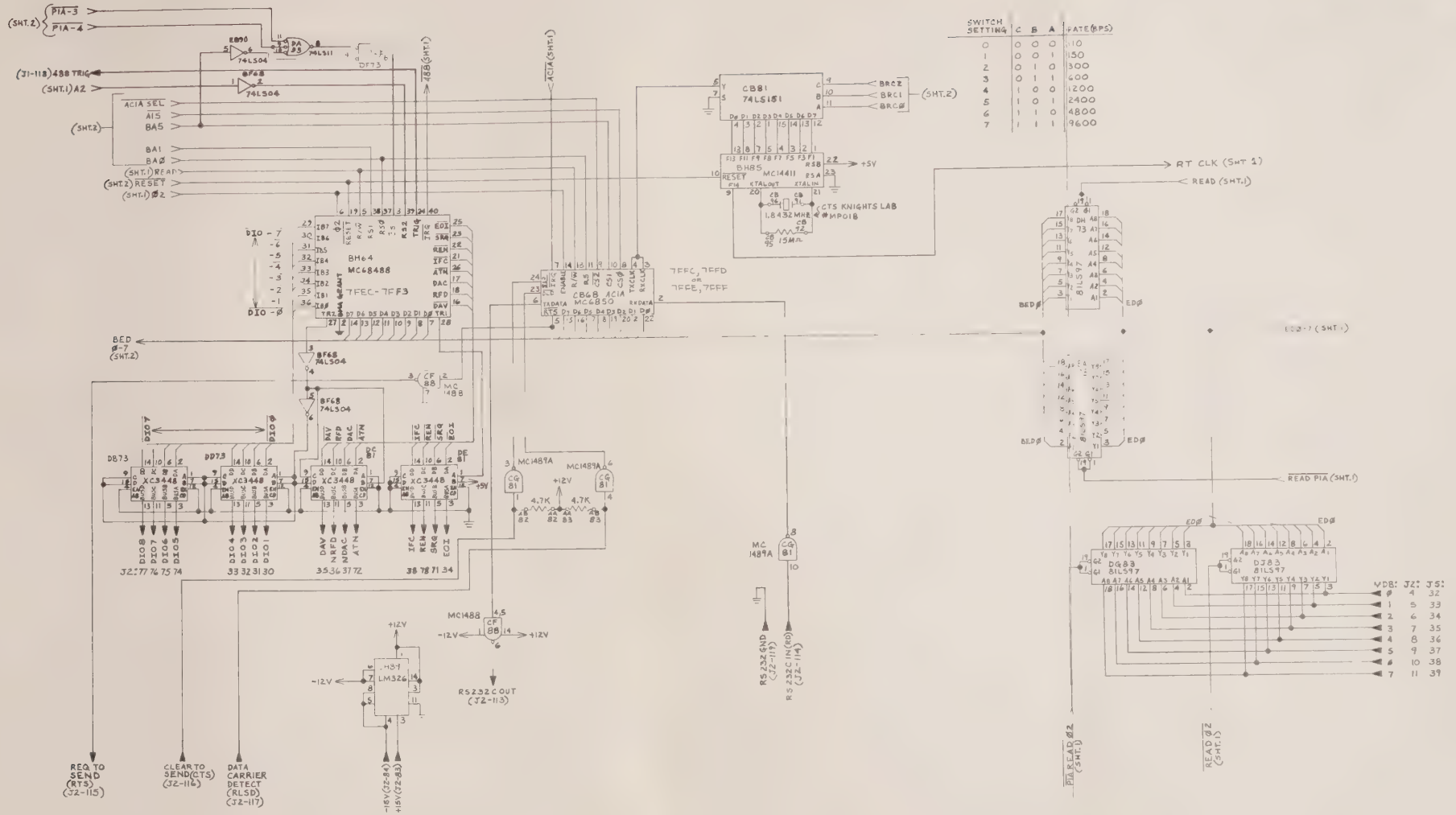


nicolet scientific



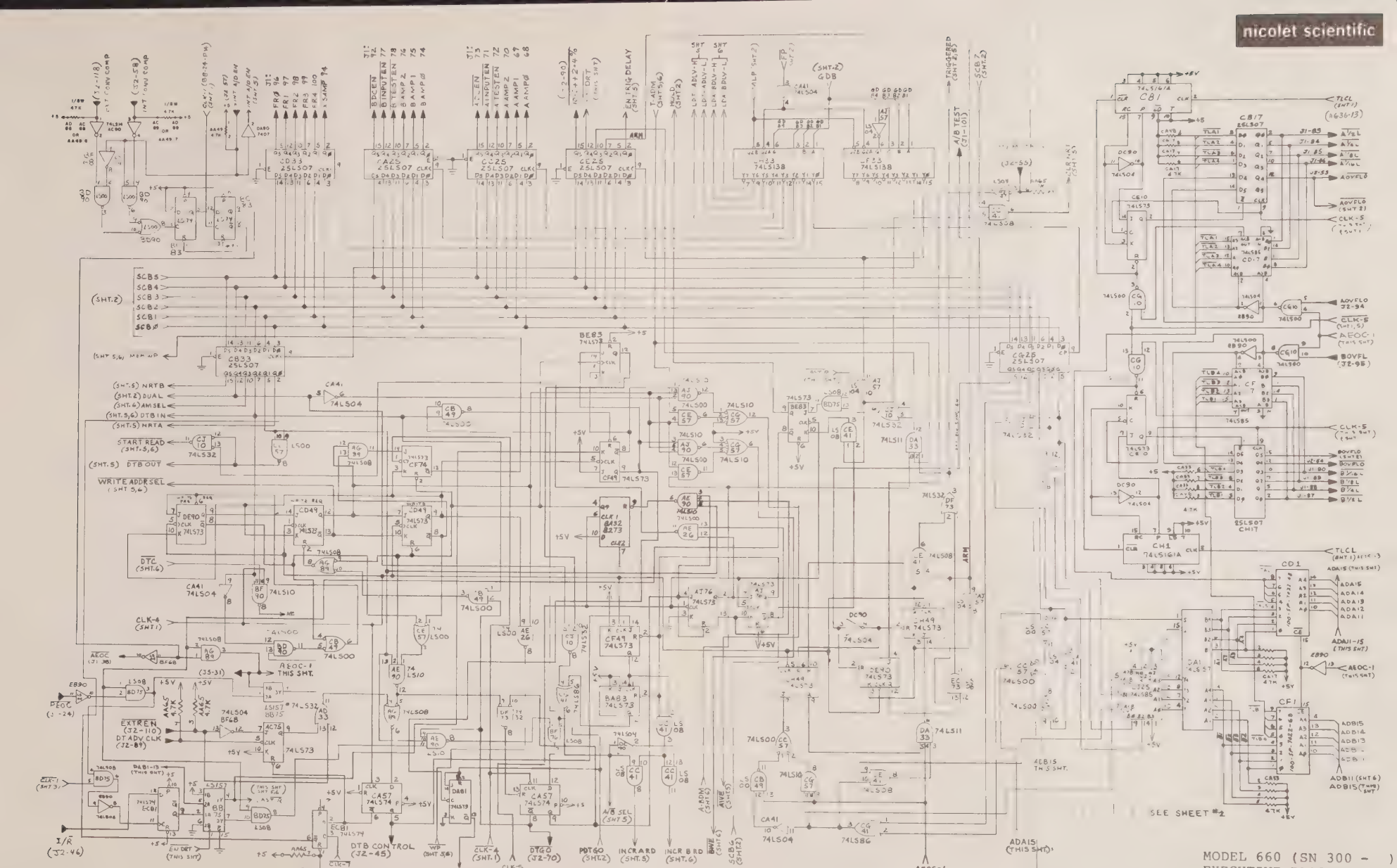
MODEL 660 (SN 300 -
EXECUTIVE PROCESSOR
100-SC-8285
SHEET 2 of 6 REV E
2-80

nicolet scientific



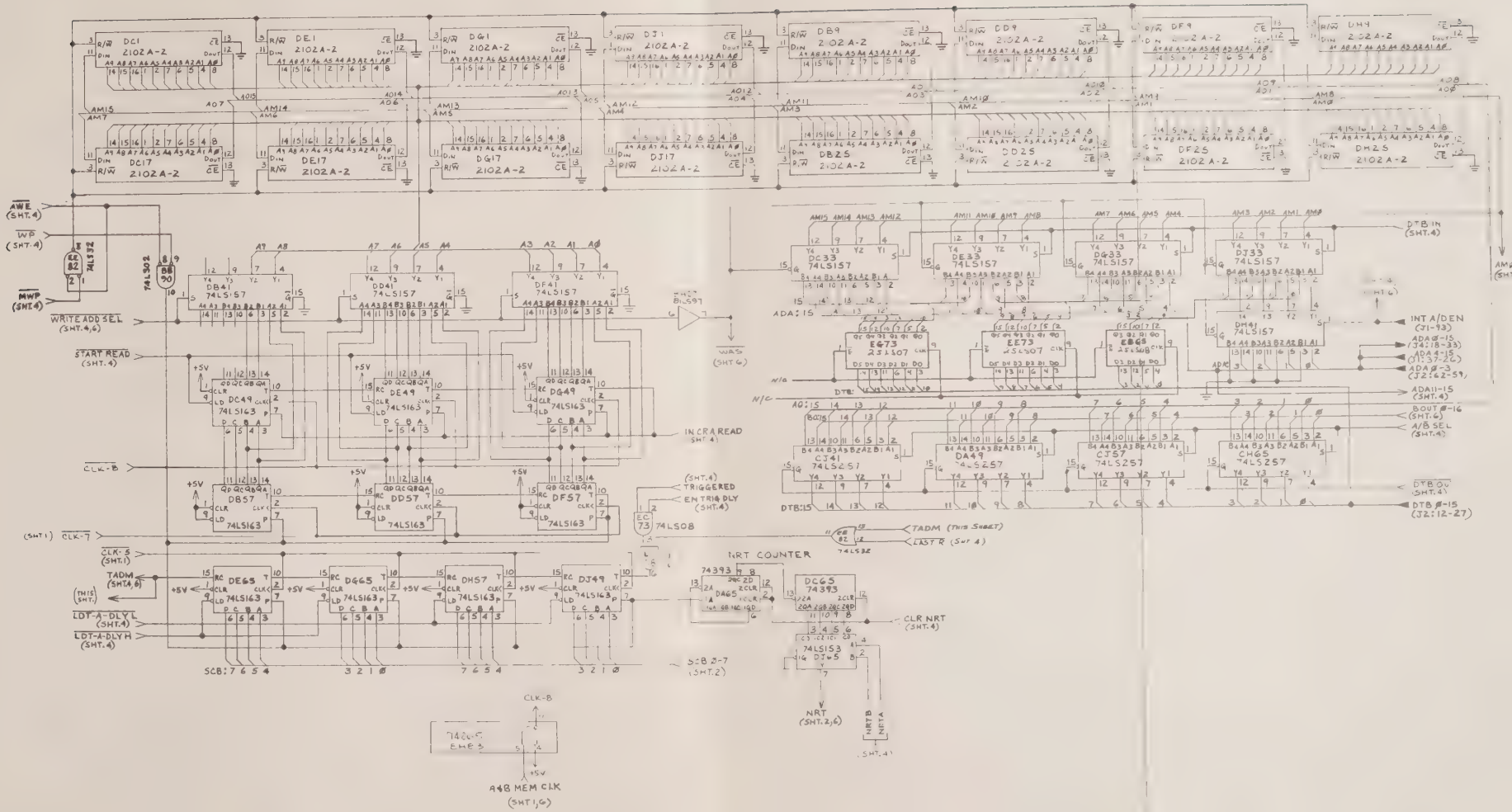
MODEL 660 (SN 300 -)
EXECUTIVE PROCESSOR
100-SC-8285
SHEET 3 of 6 REV E
2-80

nicolet scientific



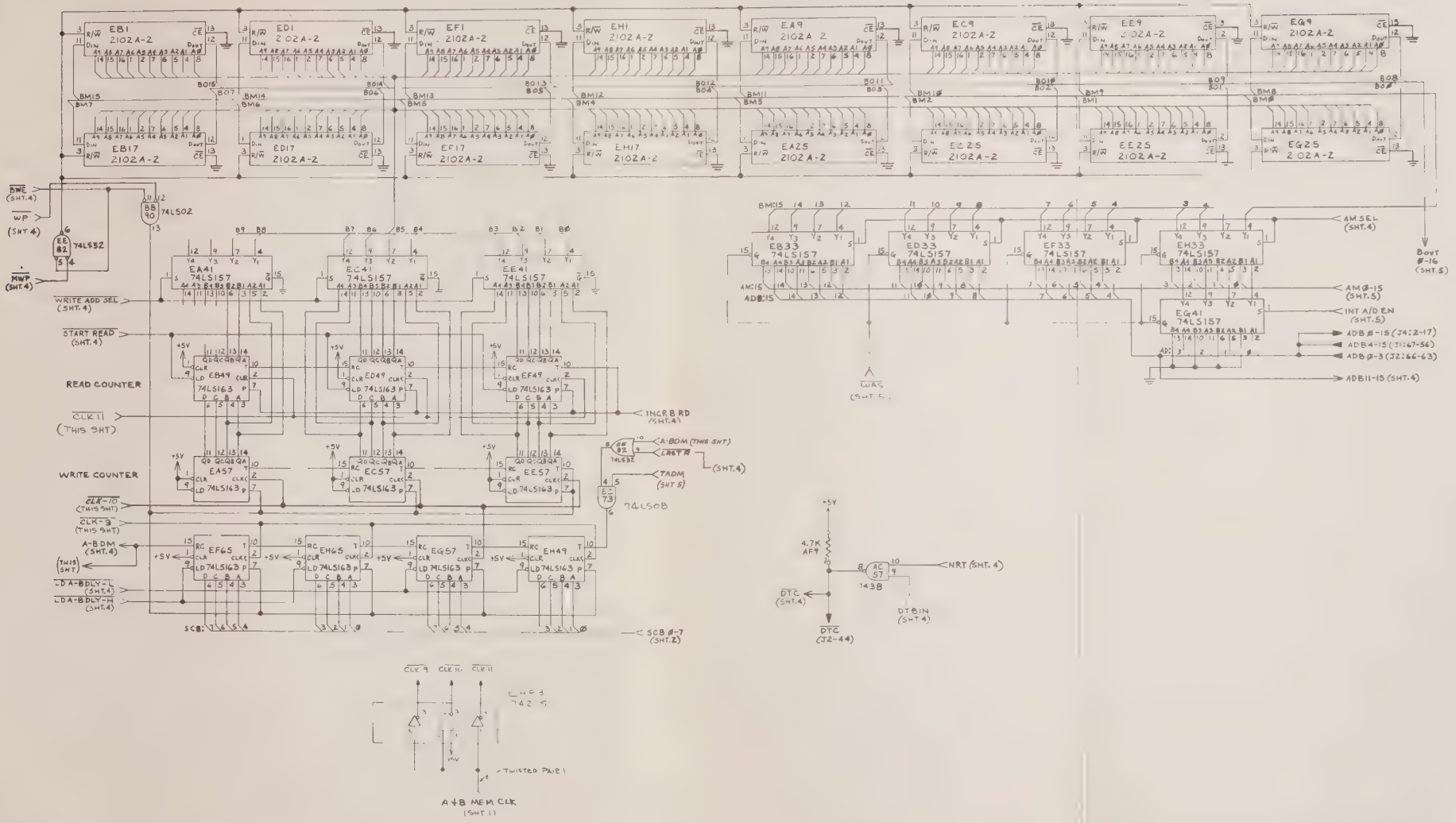
MODEL 660 (SN 300 -)
EXECUTIVE PROCESSOR
100-SC-8285
SHEET 4 of 6 REV E
2-80

nicolet scientific



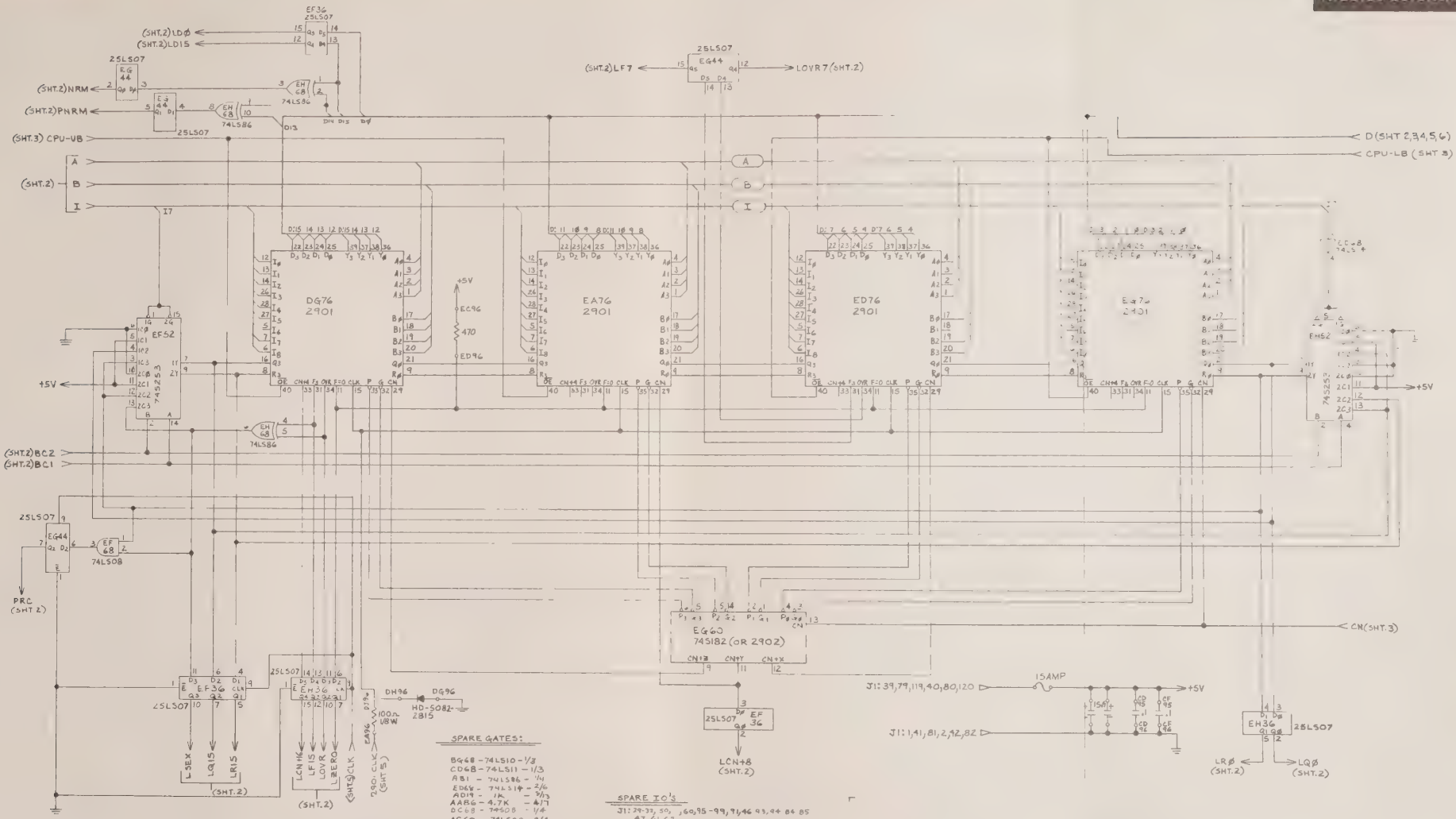
MODEL 660 (SN 300 -)
EXECUTIVE PROCESSOR
100-SC-8285
SHEET 5 of 6 REV E
2-80

nicolet scientific



MODEL 660 (SN 300 -)
EXECUTIVE PROCESSOR
100-SC-8285
SHEET 6 of 6 REV E
2-80

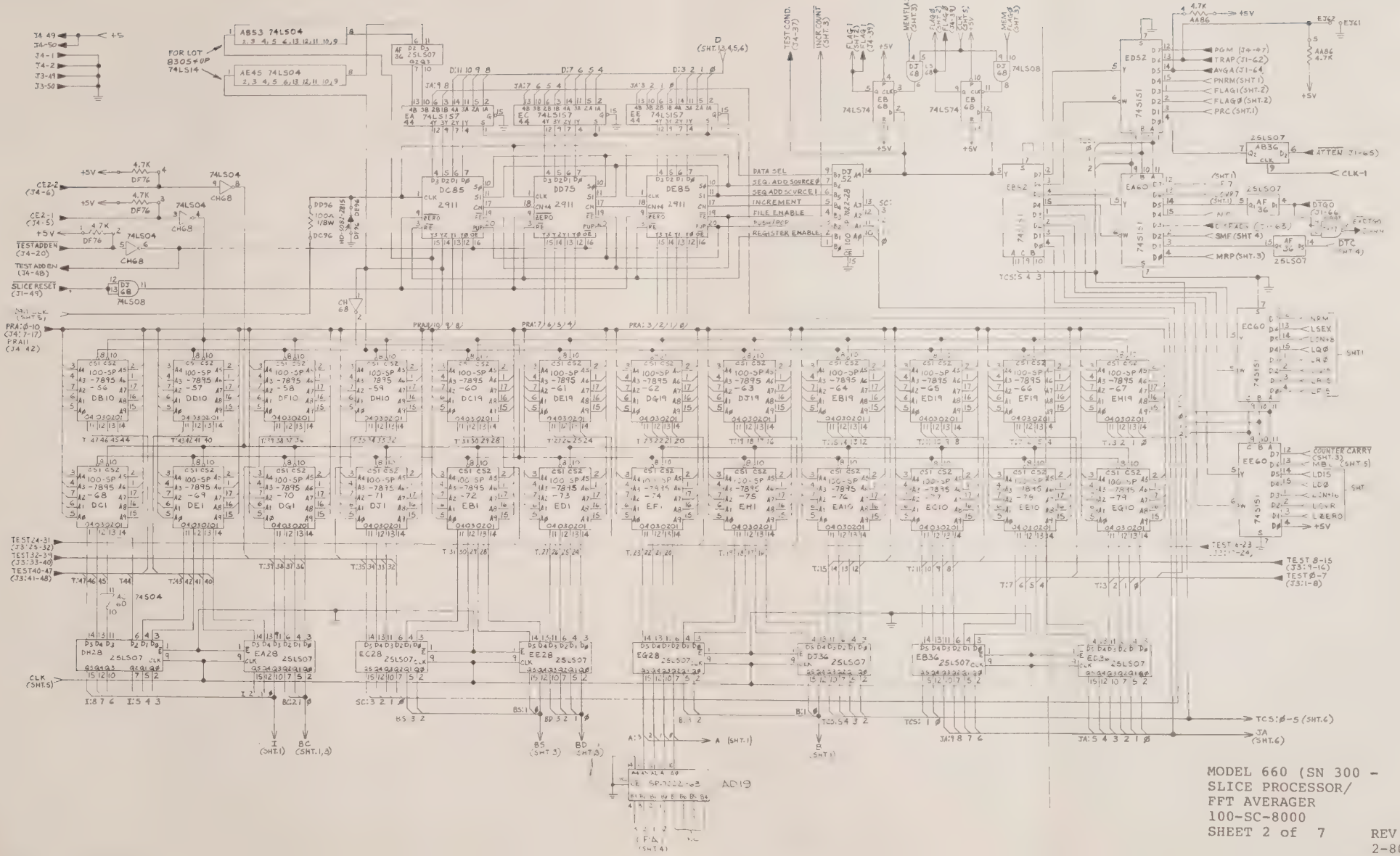
nicolet scientific



MODEL 660 (SN 300 -)
SLICE PROCESSOR/
FFT AVERAGER
100-SC-8000
SHEET 1 of 7

REV E
2-80

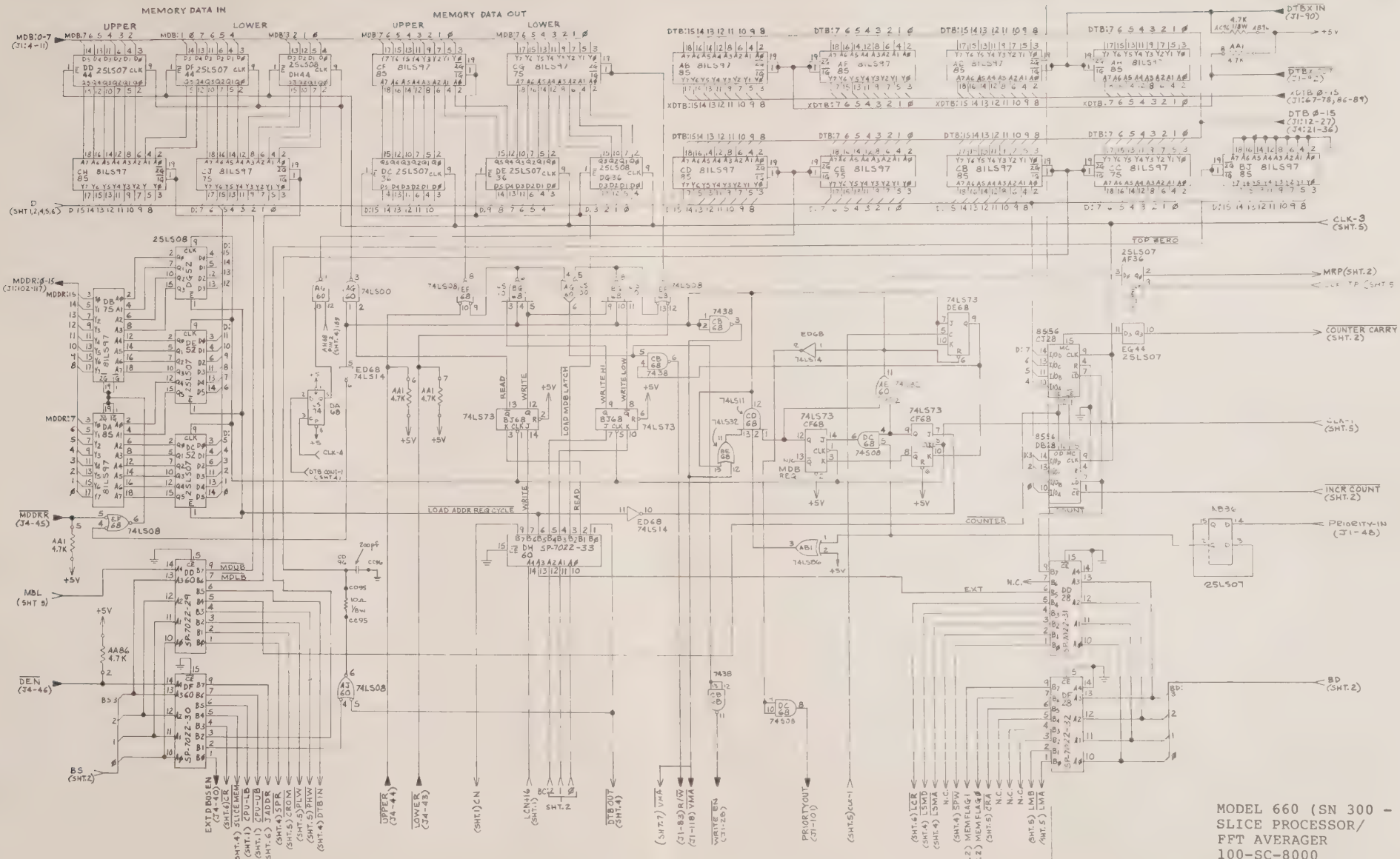
nicolet scientific



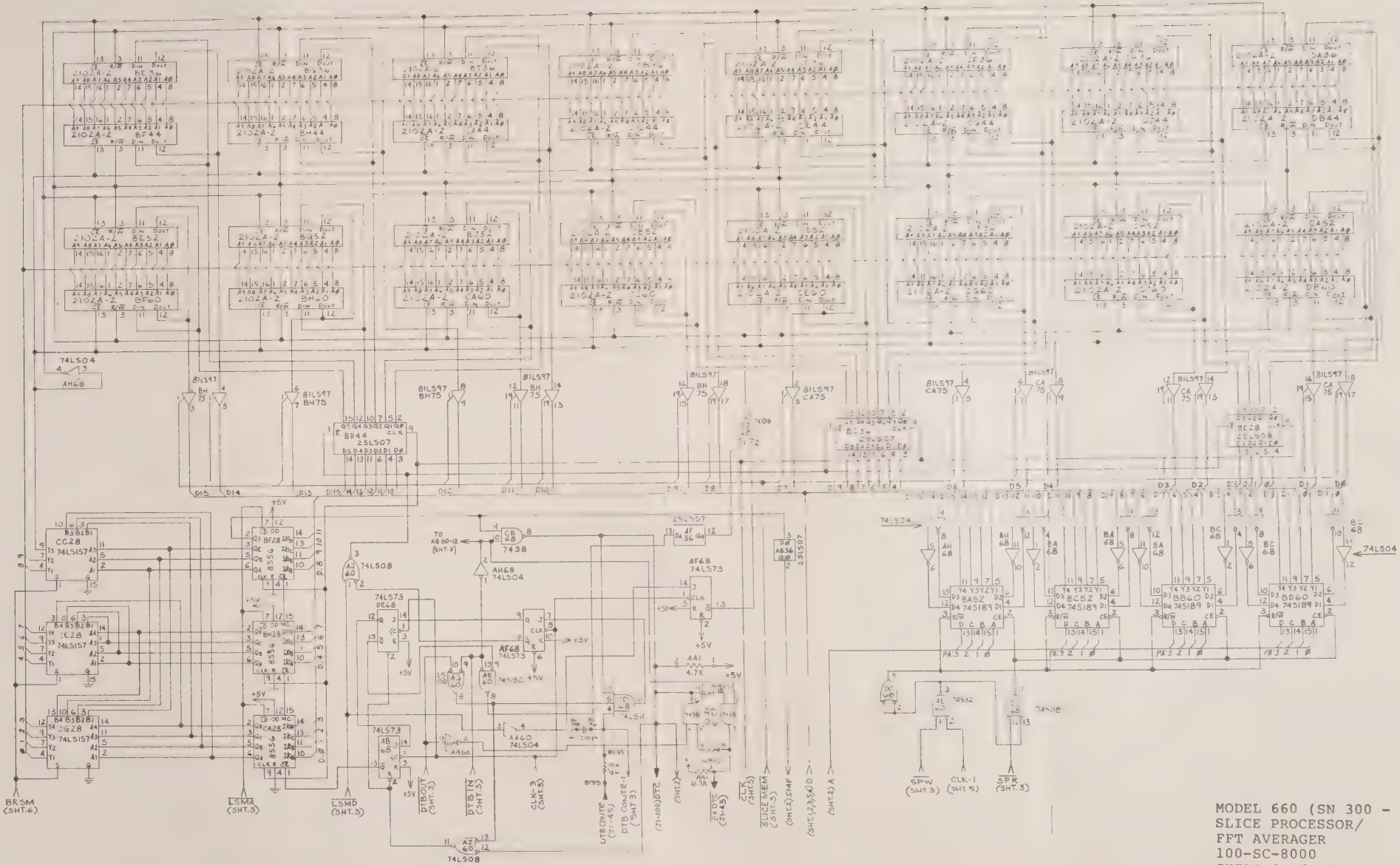
MODEL 660 (SN 300 -)
SLICE PROCESSOR/
FFT AVERAGER
100-SC-8000
SHEET 2 of 7

REV E
2-80

nicolet scientific



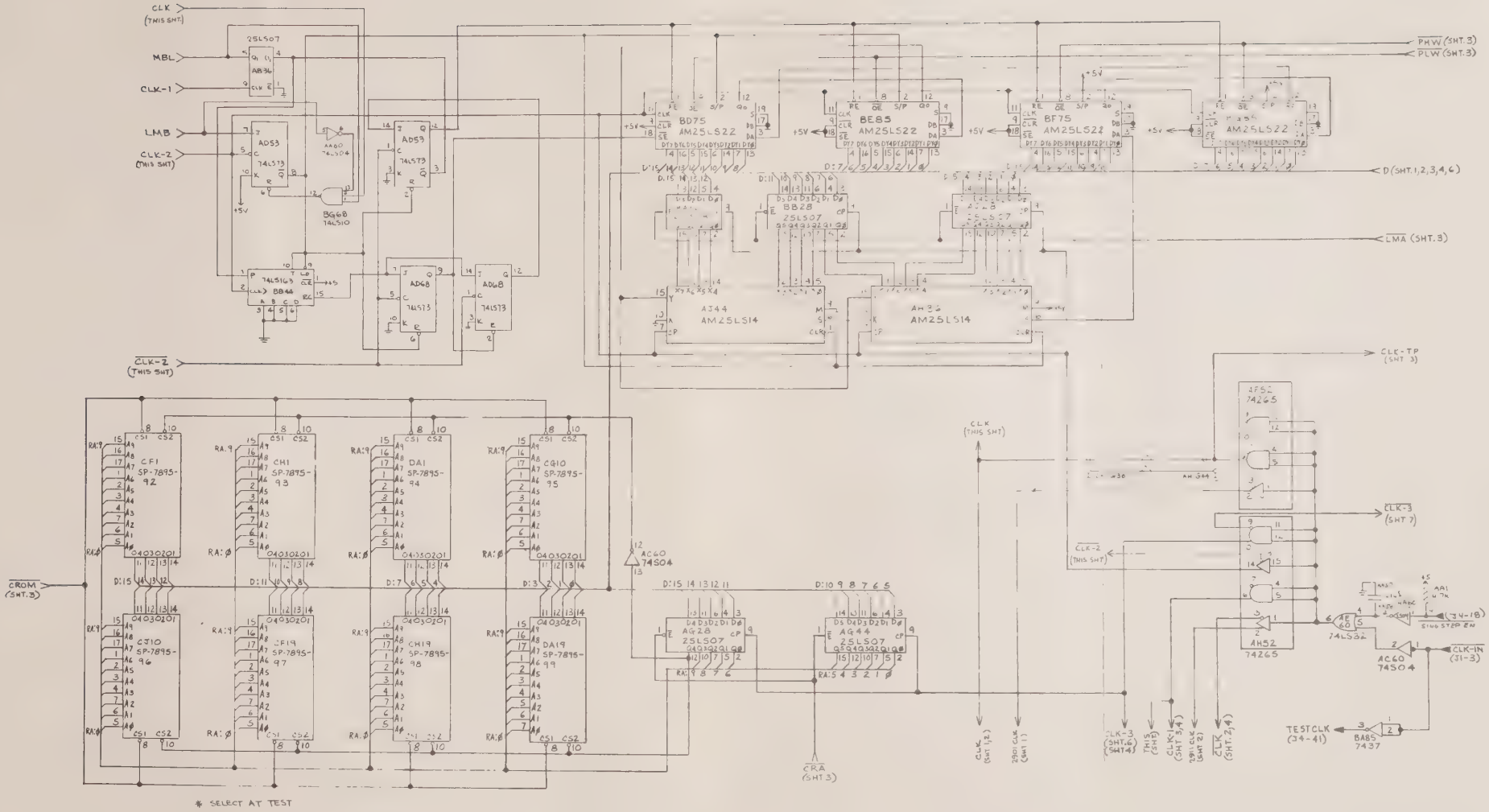
nicolet scientific



MODEL 660 (SN 300 -)
SLICE PROCESSOR/
FFT AVERAGER
100-SC-8000
SHEET 4 of 7

REV E
2-80

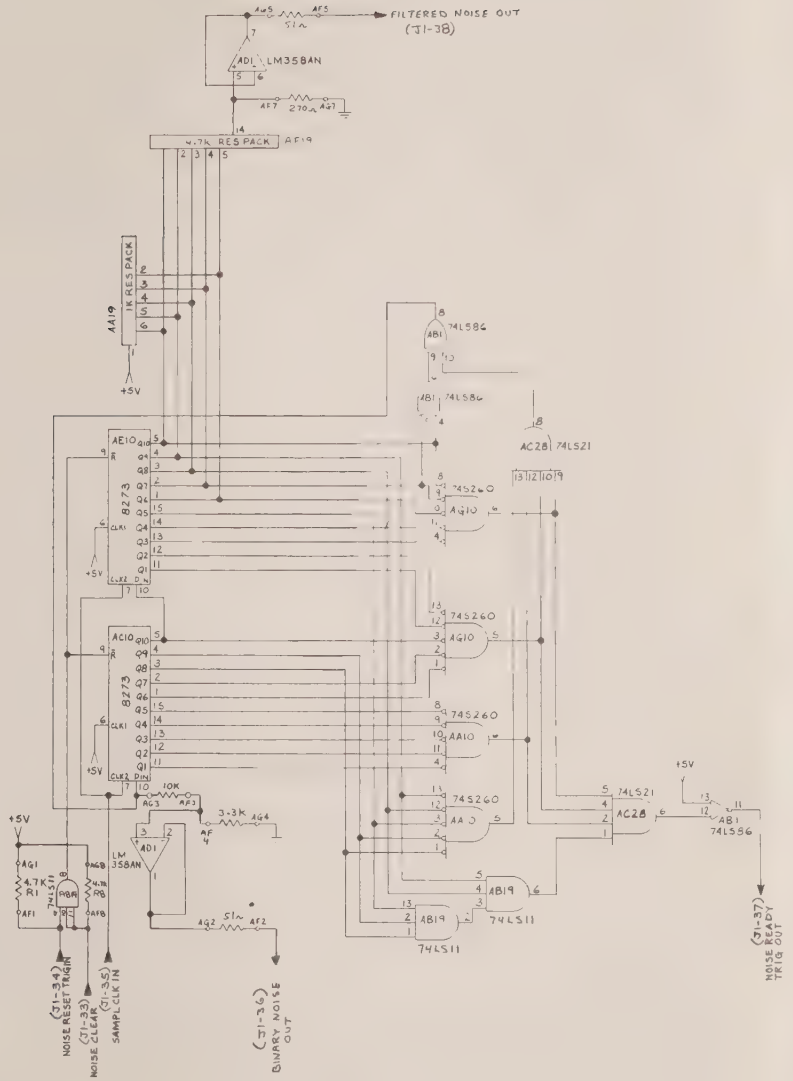
nicolet scientific

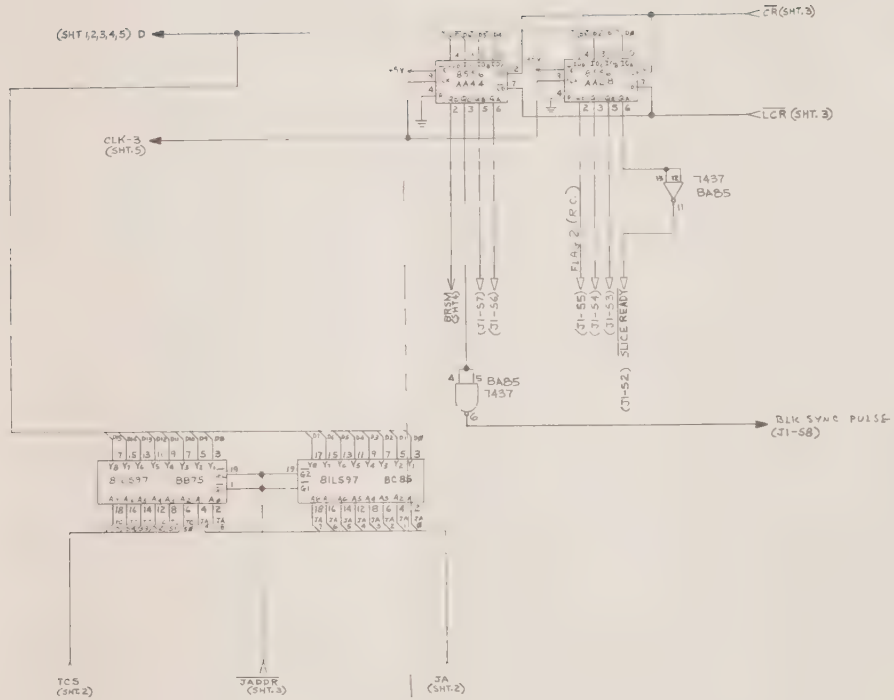


MODEL 660 (SN 300 -)
SLICE PROCESSOR/
FFT AVERAGER
100-SC-8000
SHEET 5 of 7

REV E
2-80

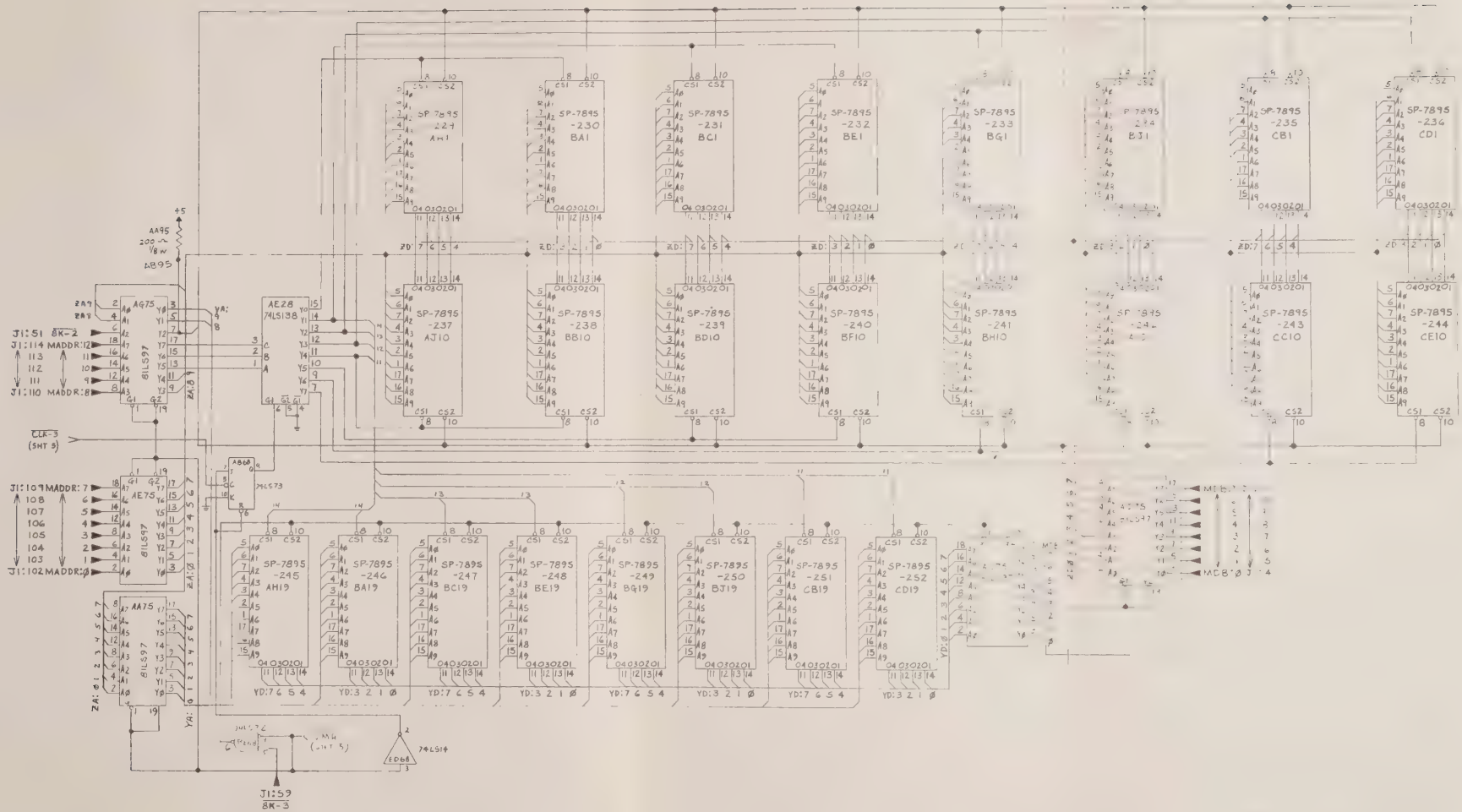
nicolet scientific





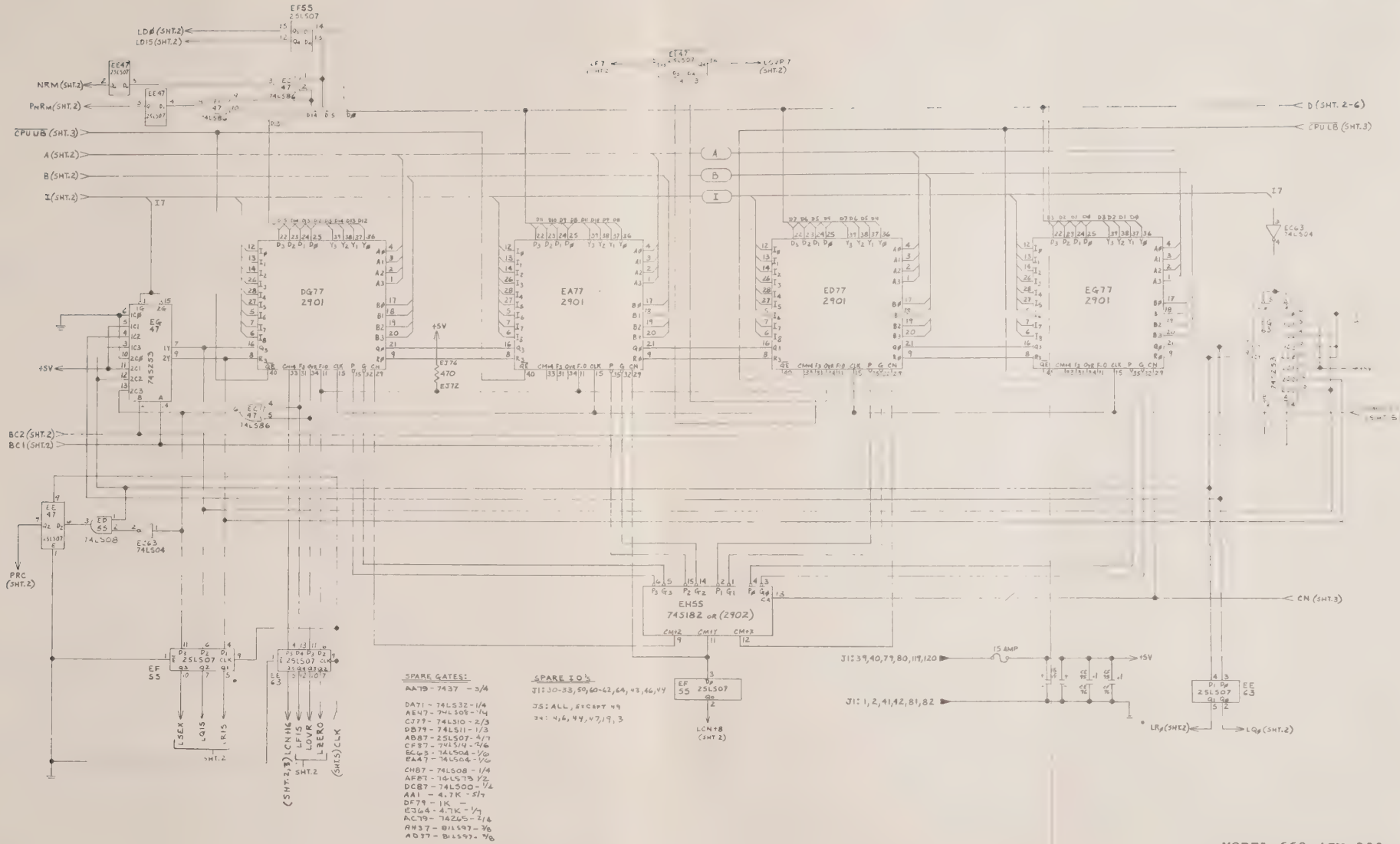
MODEL 660 (SN 300 -)
SLICE PROCESSOR/
FFT AVERAGER
100-SC-8000
SHEET 6 of 7 REV E
2-80

nicolet scientific



MODEL 660 (SN 300 -)
SLICE PROCESSOR/
FFT AVERAGER
100-SC-8000
SHEET 7 of 7 REV E
2-80

nicolet scientific



SPARE GATES:

AA79 - 7437 - 3/4

DA71 - 74LS32 - 1/4
AE47 - 74LS09 - 1/4
CJ79 - 74LS10 - 2/3
DB79 - 74LS11 - 1/3
AB87 - 25LS07 - 4/7
CF87 - 74LS14 - 6/6
EG63 - 74LS04 - 1/6
EA47 - 74LS04 - 1/6
CH87 - 74LS08 - 1/4
AF87 - 74LS73 1/2
DG87 - 74LS00 - 1/4
AA1 - 4.7K - 5/7
DF79 - 1K -
EJ64 - 4.7K - 1/7
AC79 - 74245 - 2/4
AH37 - BIL597 - 3/6
AD37 - BIL597 - 4/6

SPARE I O's

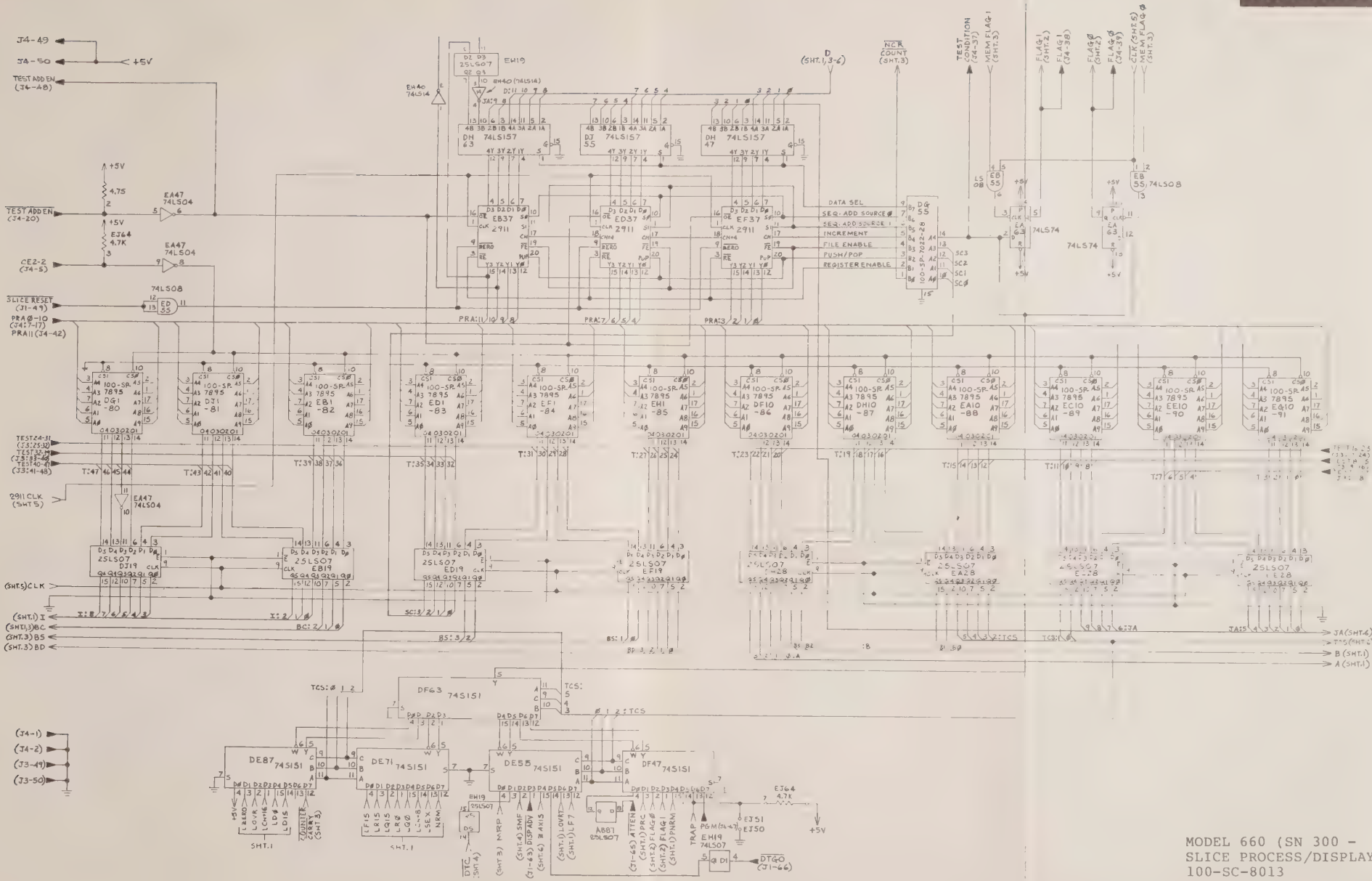
J1: 30-33, 50, 60-62, 64, 43, 46, 44

J3: ALL, EXCEPT 49

J4: 4, 6, 44, 47, 7, 9, 3

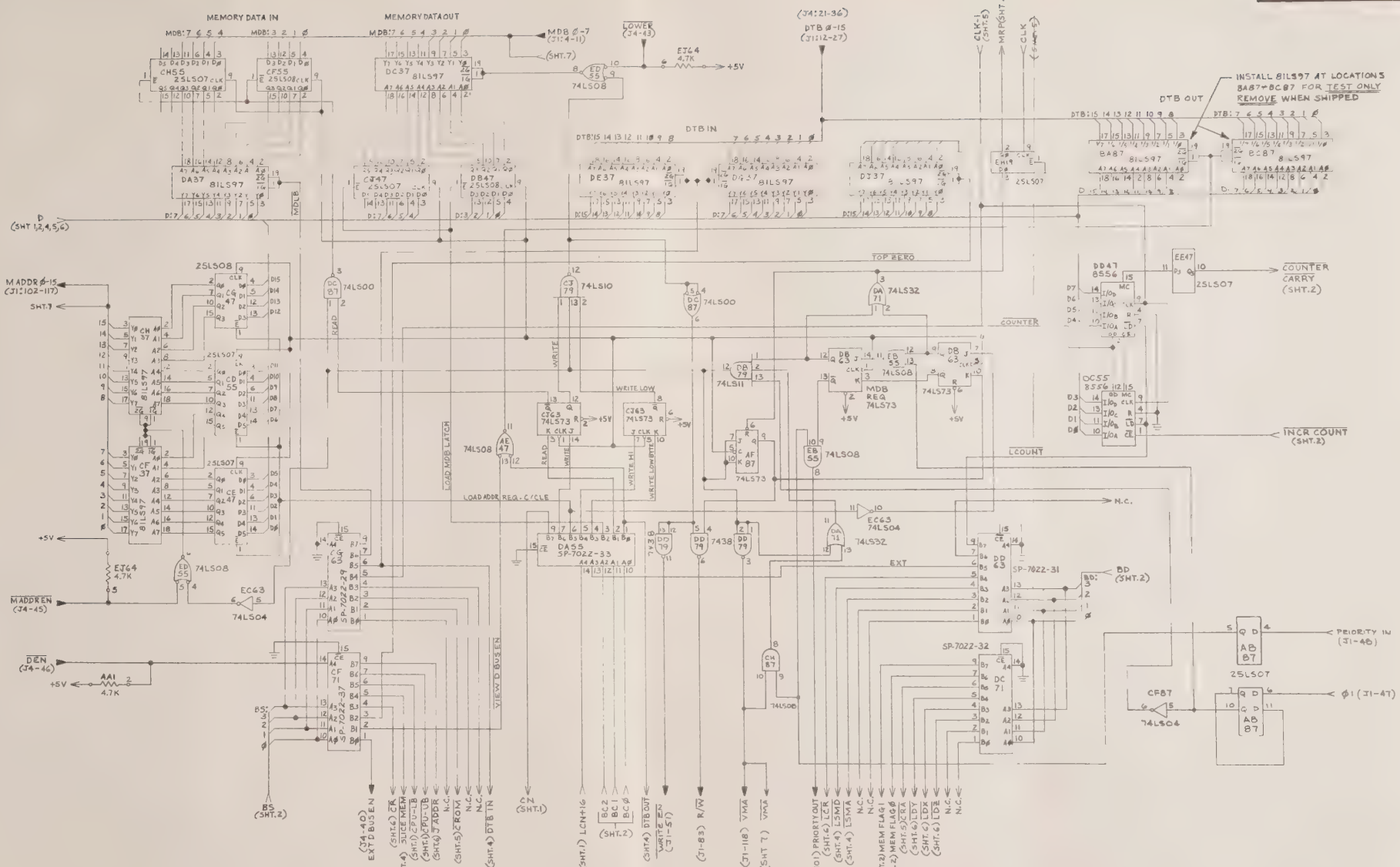
MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 1 of 8 REV E
2-80

nicolet scientific



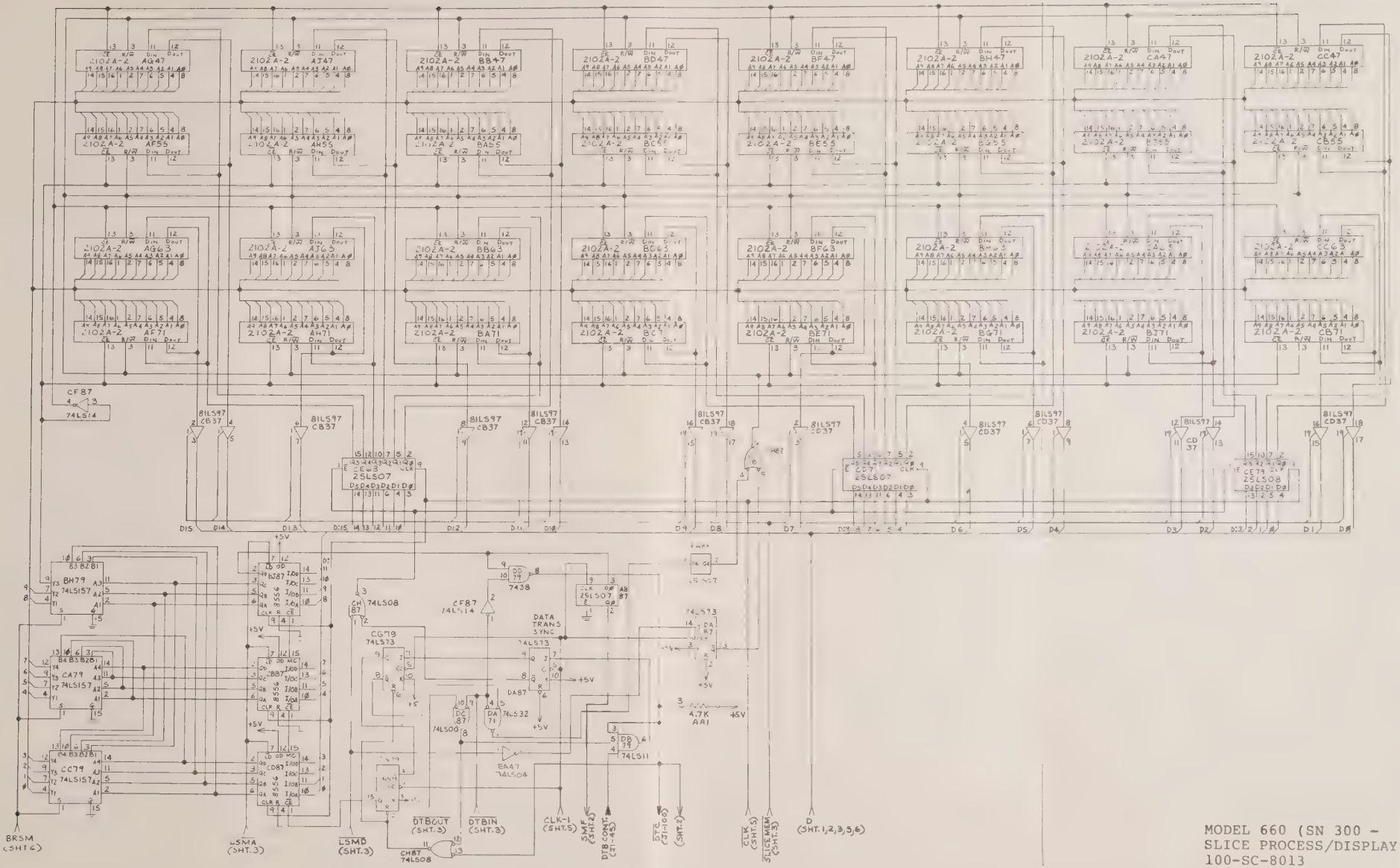
MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 2 of 8 REV E
2-80

nicolet scientific



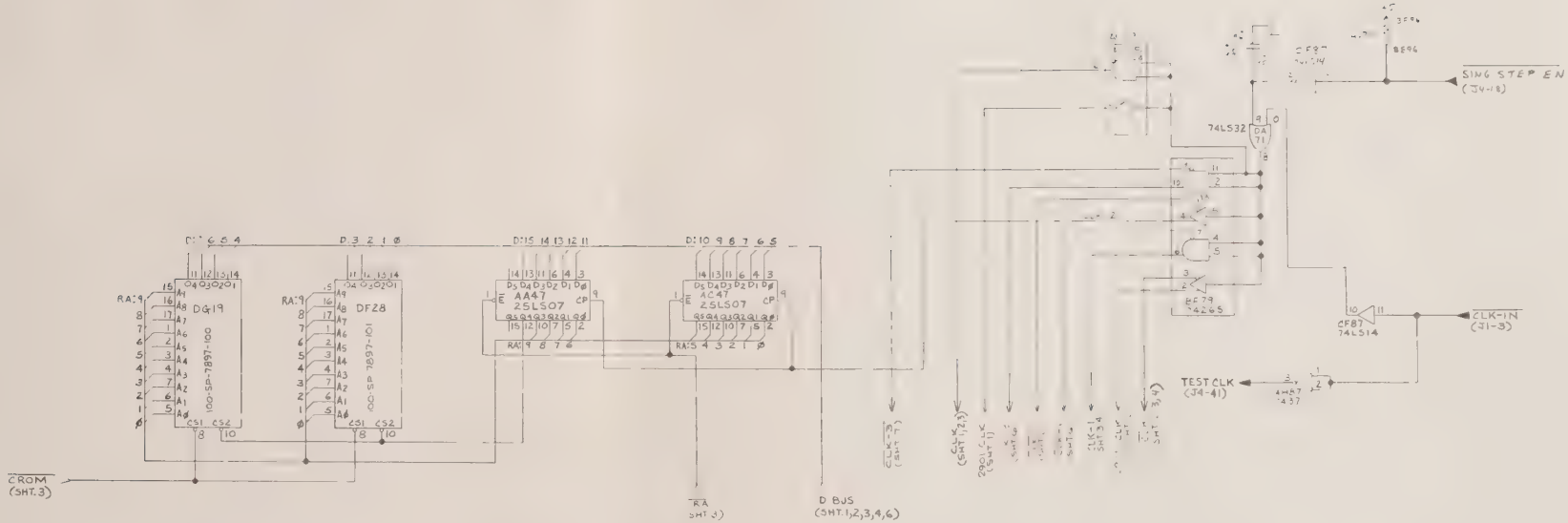
MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 3 of 8 REV E
2-80

nicolet scientific



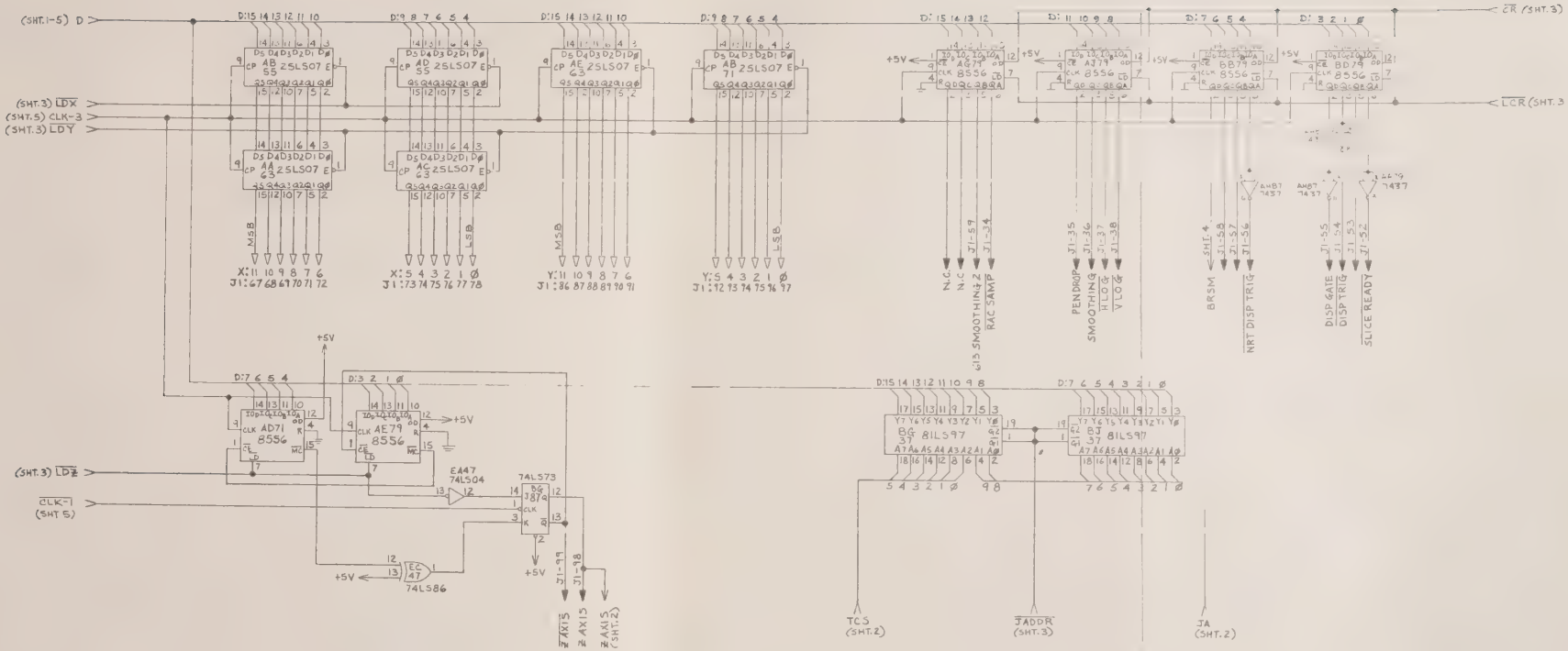
MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 4 of 8 REV E
2-80

nicolet scientific

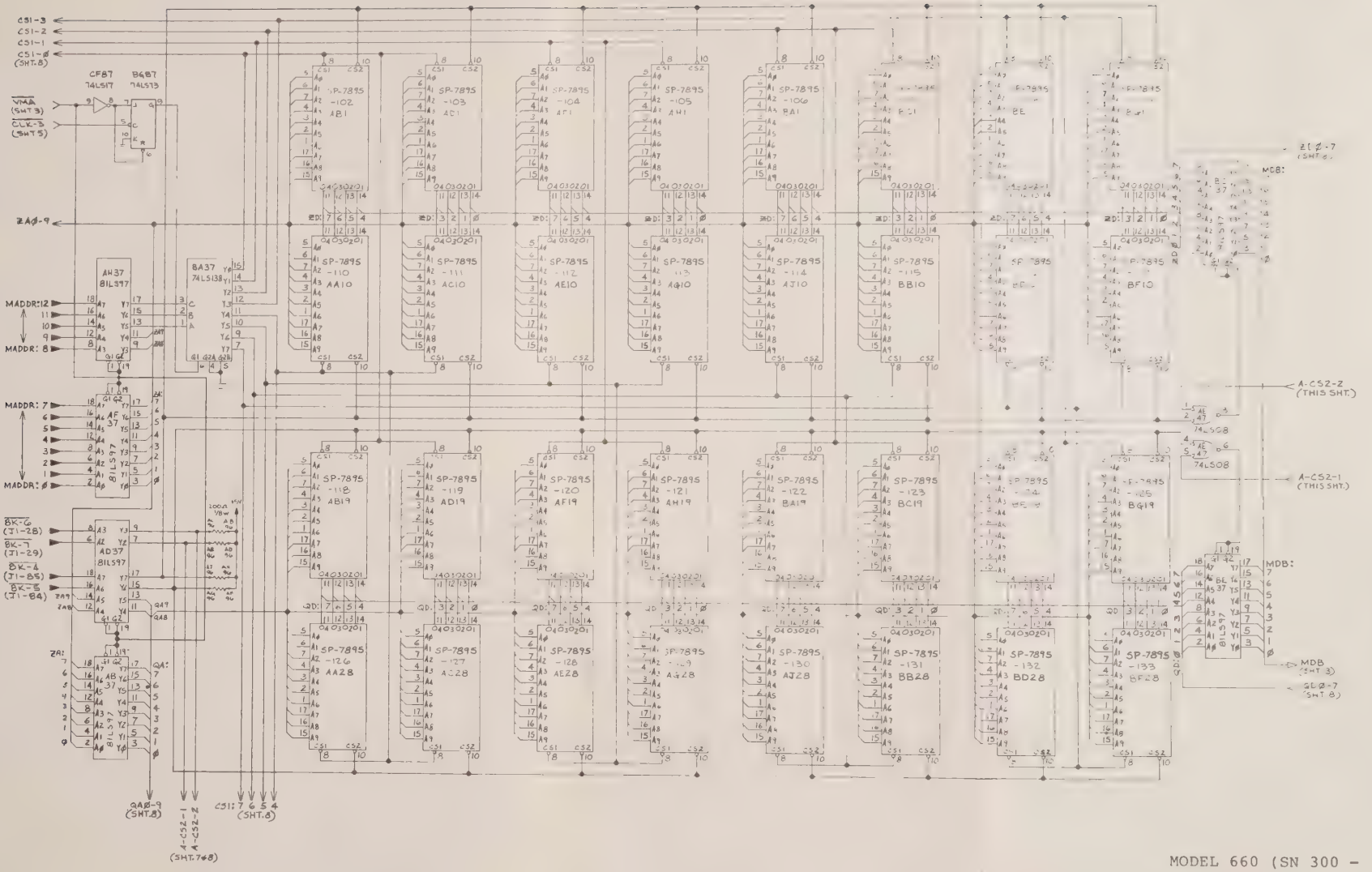


MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 5 of 8 REV E
2-80

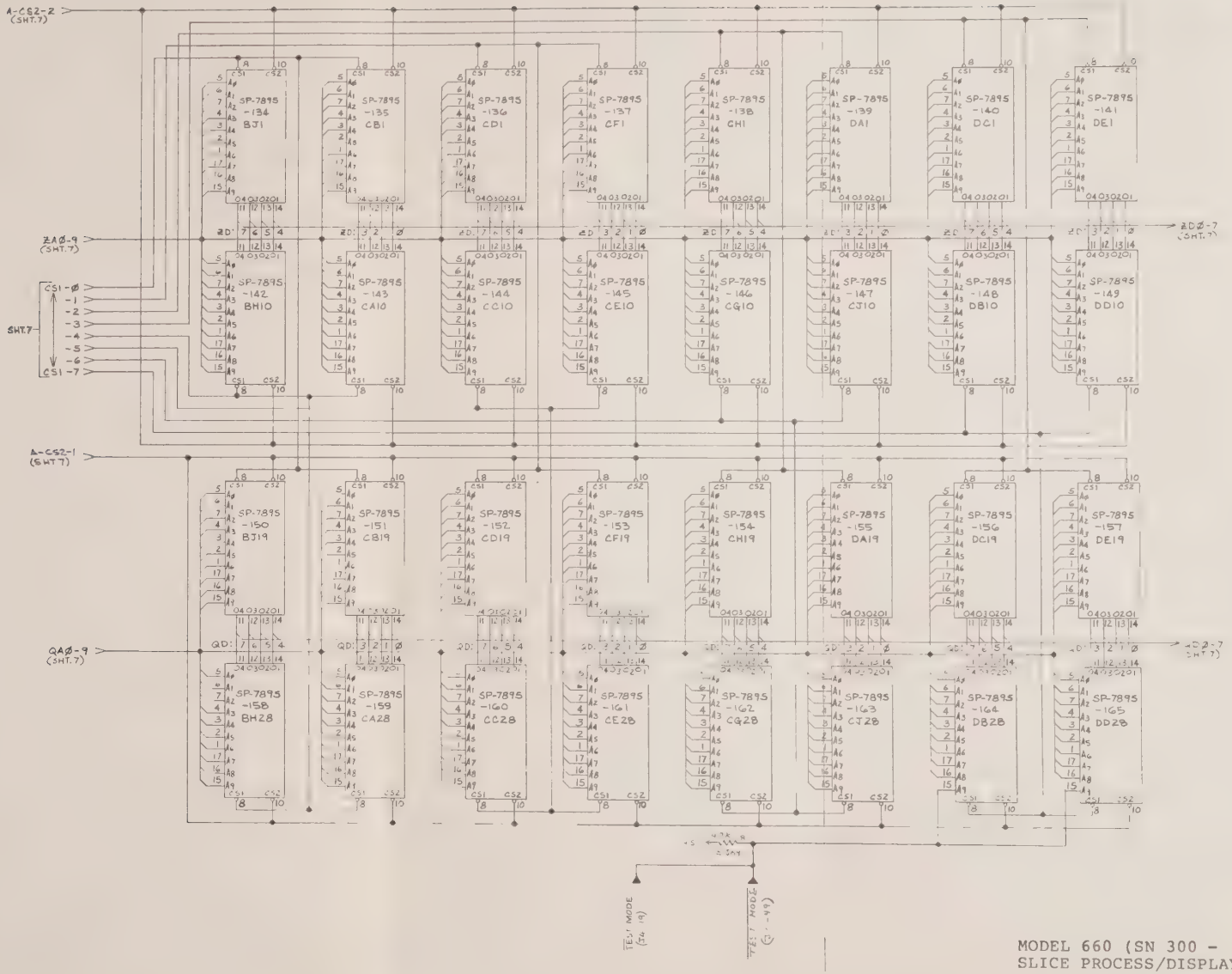
nicolet scientific



MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 6 of 8 REV E
2-80

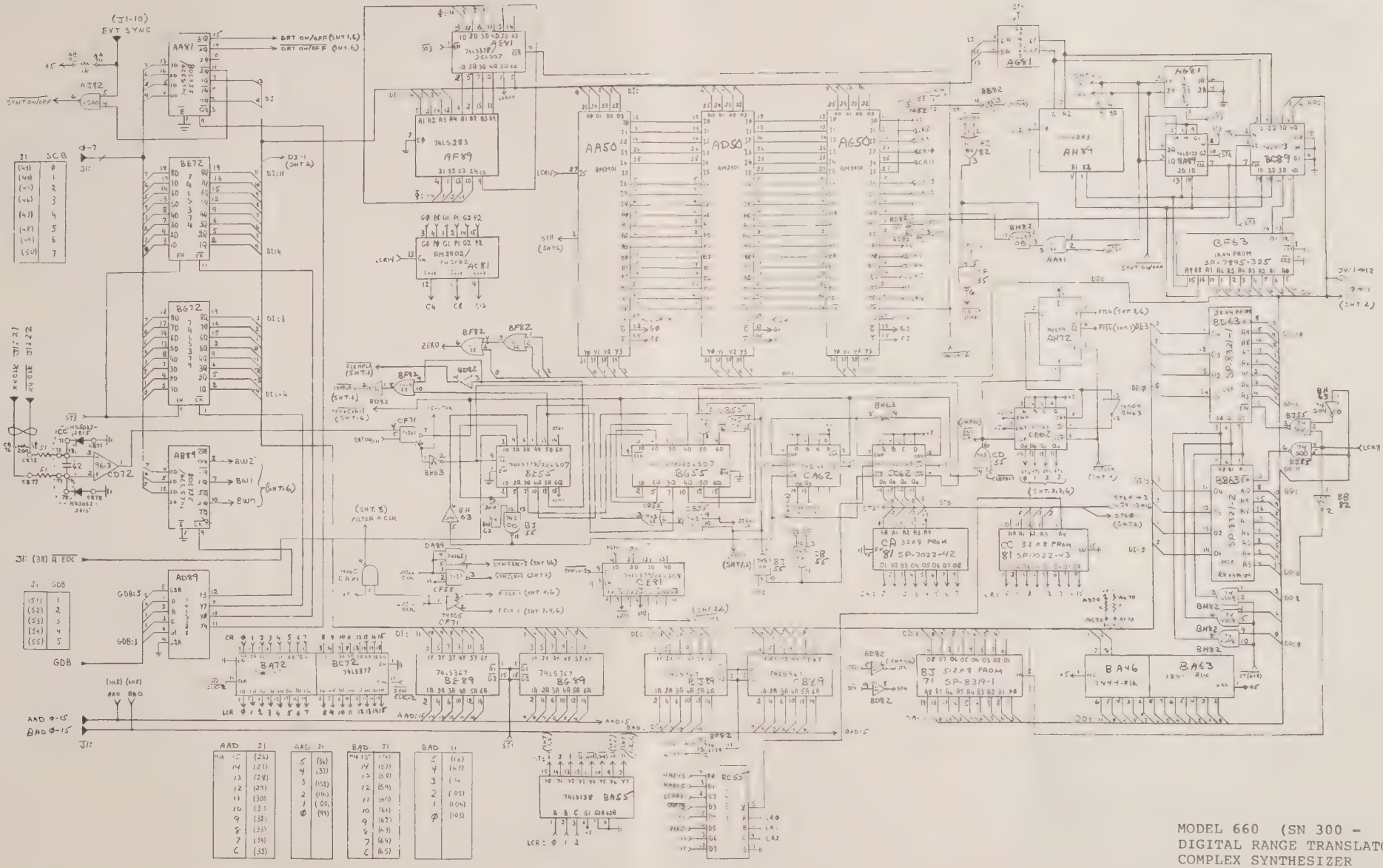
nicolet scientific

MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 7 of 8 REV E
2-80

nicolet scientific



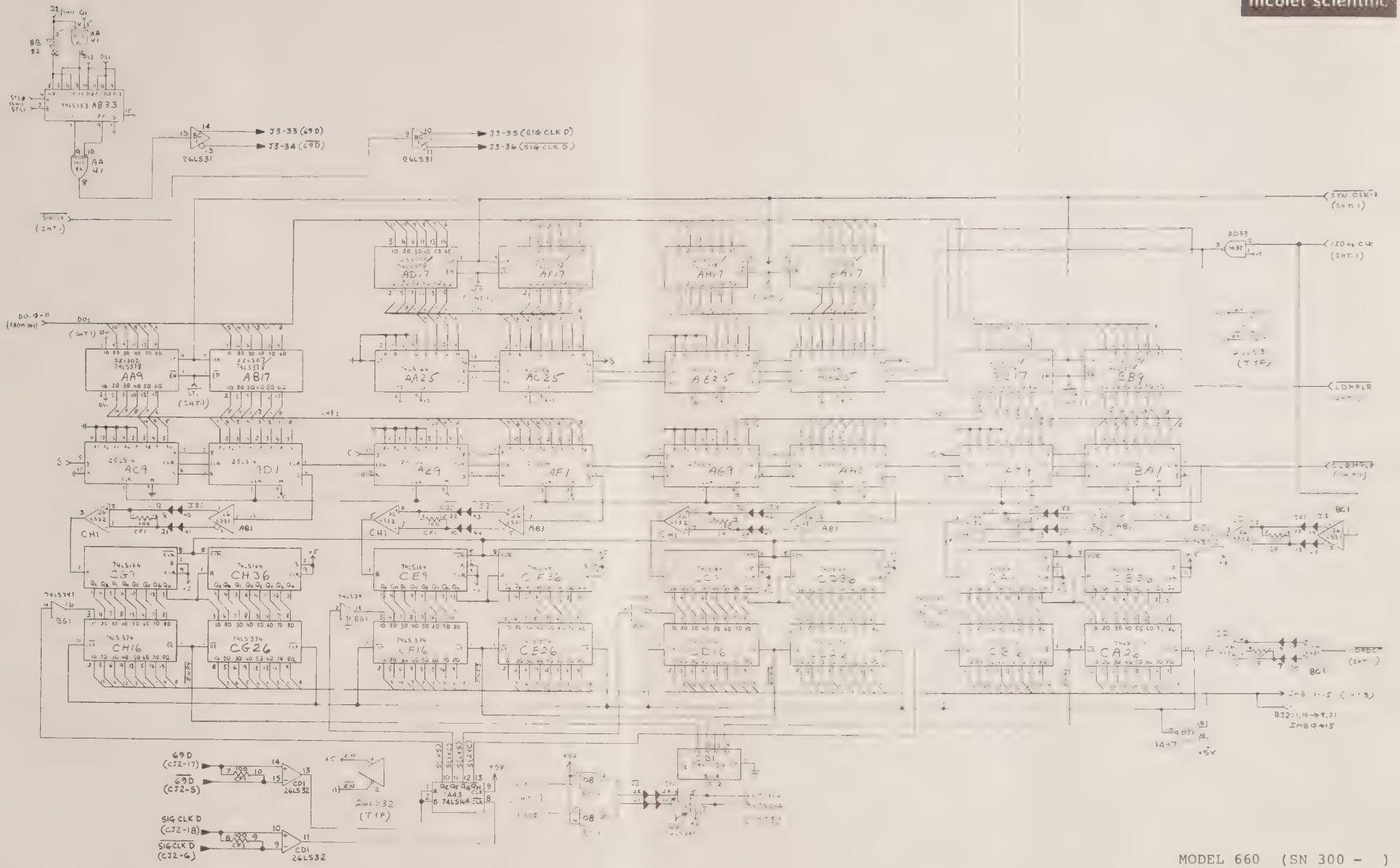
MODEL 660 (SN 300 -)
SLICE PROCESS/DISPLAY
100-SC-8013
SHEET 8 of 8 REV E
2-80

nicolet scientific



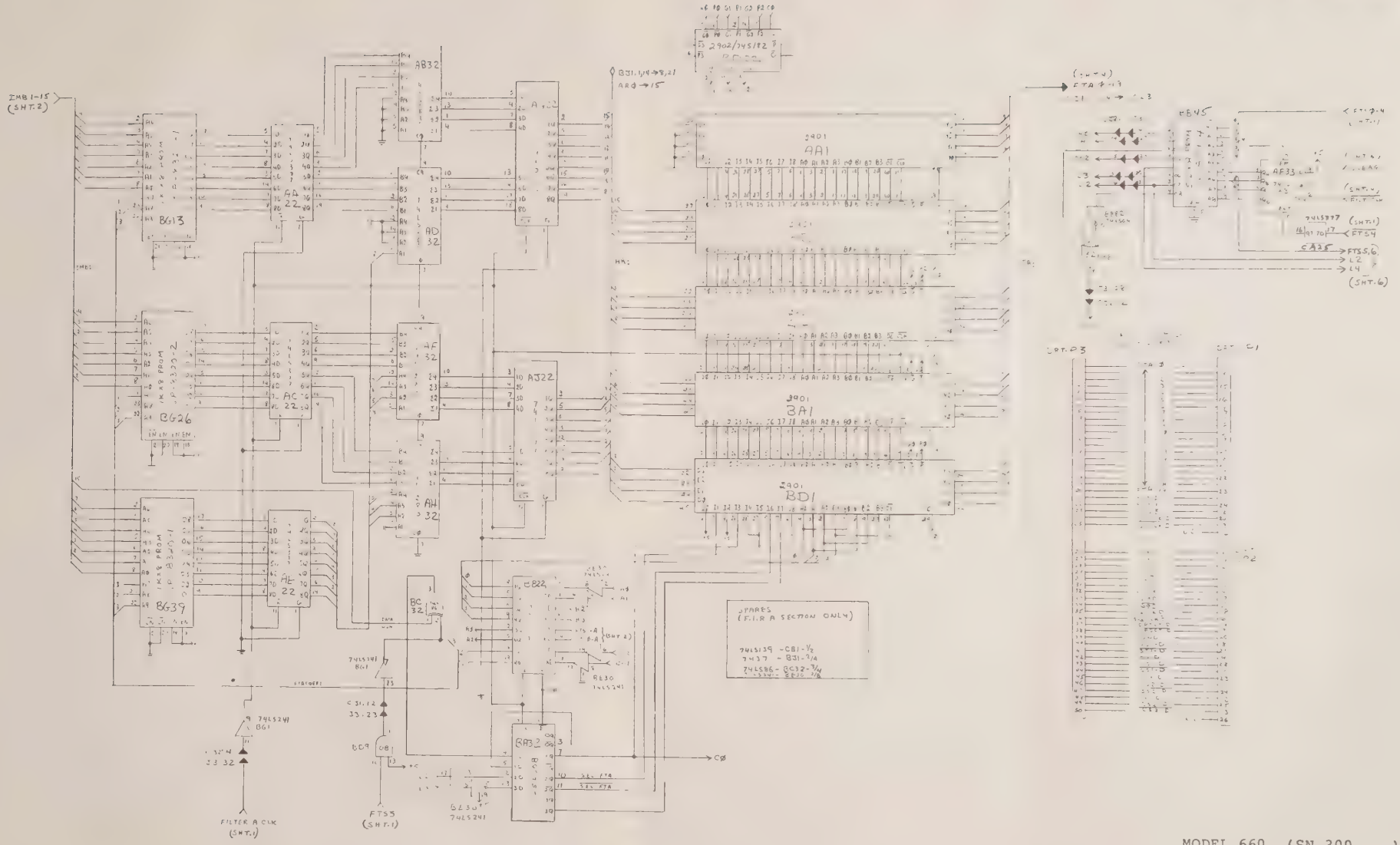
MODEL 660 (SN 300 -)
DIGITAL RANGE TRANSLATOR
COMPLEX SYNTHESIZER
100-SC-8314
SHEET 1 of 6 REV E
2-80

nicolet scientific



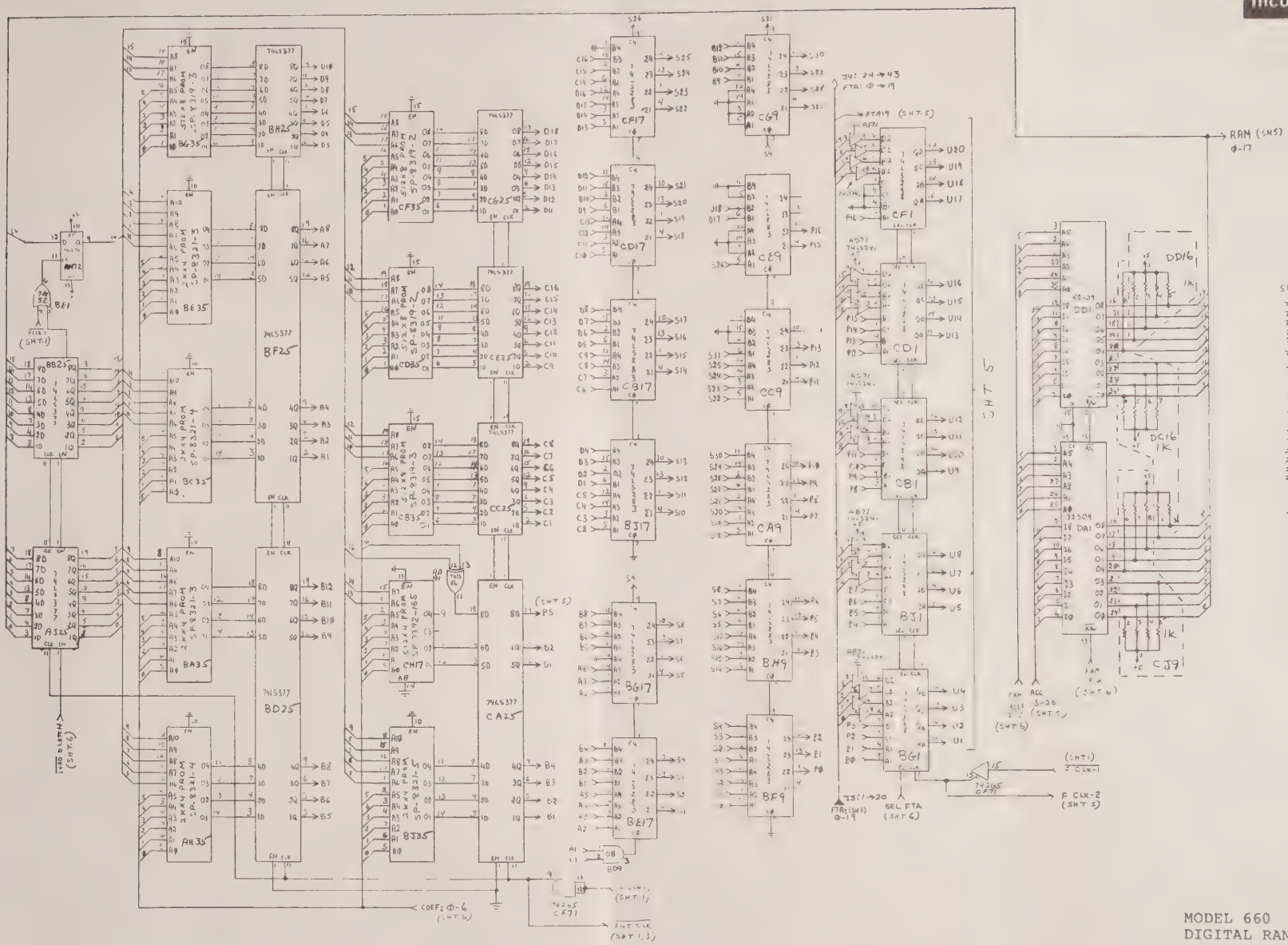
MODEL 660 (SN 300 -)
DIGITAL RANGE TRANSLATOR
INPUT MULTIPLIER
100-SC-8314
SHEET 2 of 6 REV E
2-80

nicolet scientific

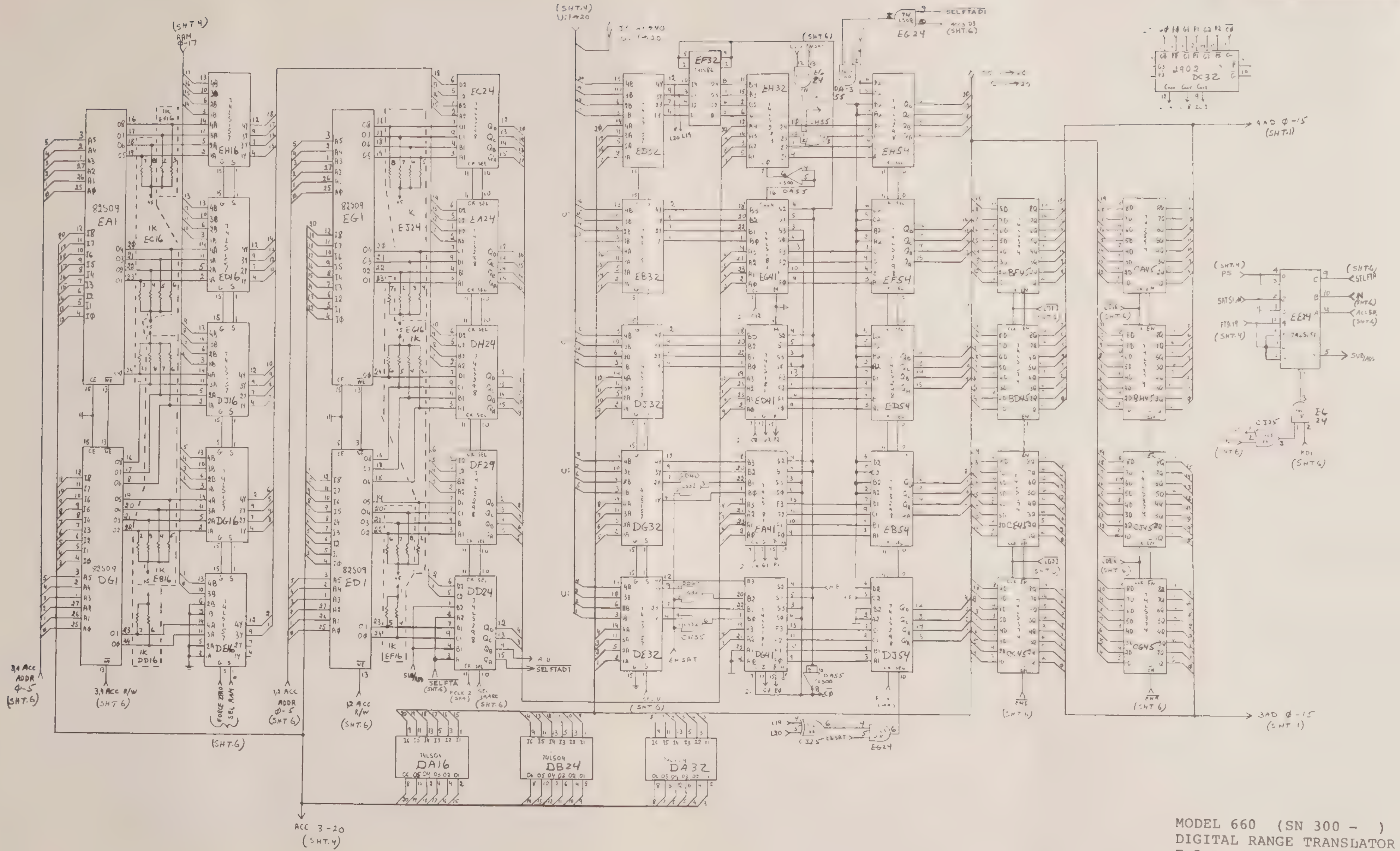


MODEL 660 (SN 300 -)
DIGITAL RANGE TRANSLATOR
F.I.R. (A) SECTION
100-SC-8314
SHEET 3 of 6 REV E
2-80

nicolet scientific

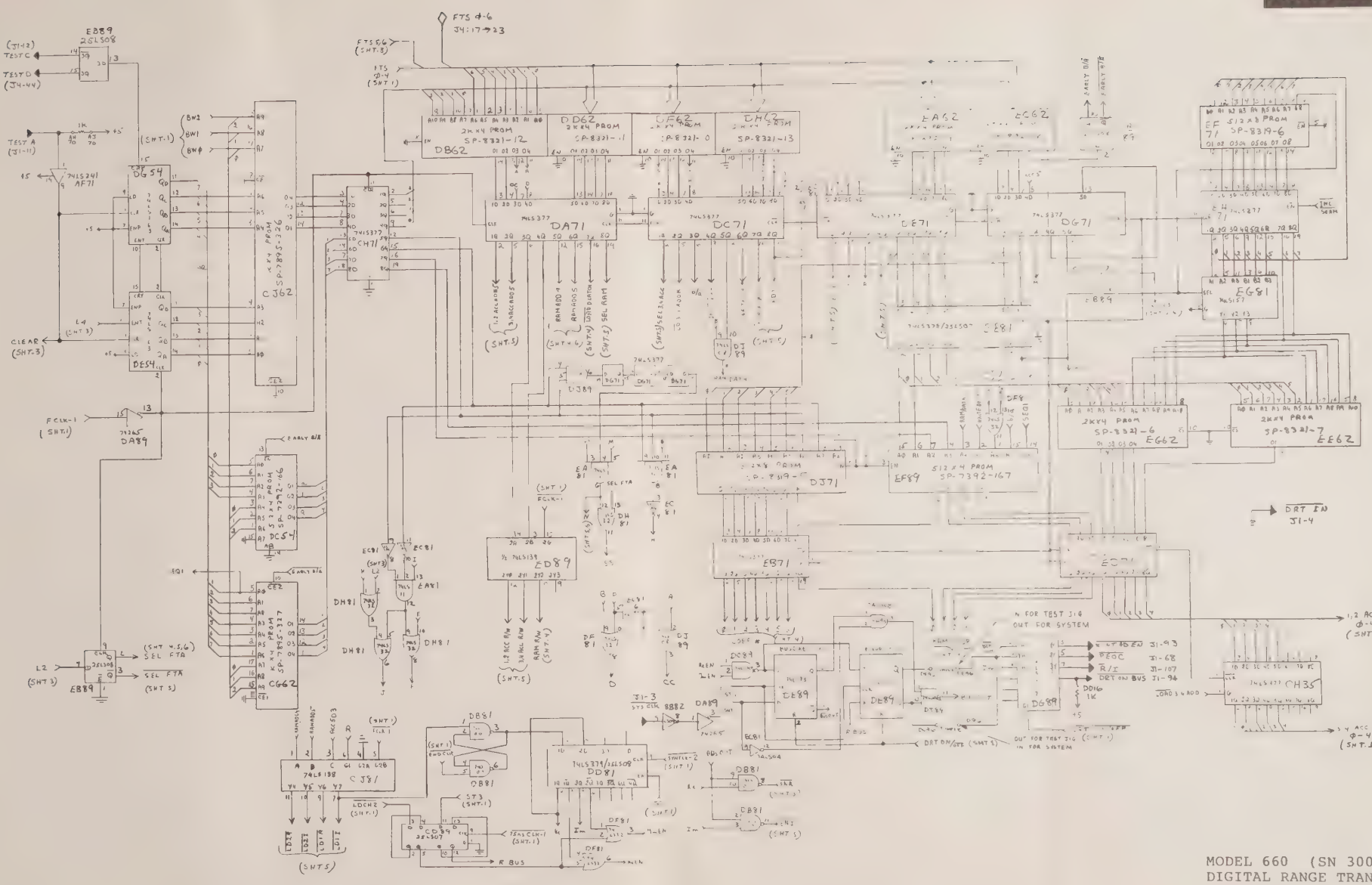


nicolet scientific


82S09
EA1
EG1
DG1
ED1
EH16
ED16
DJ16
DG16
DE16
EC24
EA24
DH24
DF29
DD24
ED32
EB32
DJ32
DG32
DE32
EF32
EH32
EG41
ED41
EA41
DG41
EH54
EF54
ED54
EB54
DJ54
BF45
BD45
CE45
CK45
CA45
BH45
CJ45
CG45
2902
DC32
EE24
DA16
DB24
DA32
74LS04
RAM Φ-17
(SHT 4)
(SHT.4)
U:1→20
SELFTAD1
EG24
AAD Φ-15
(SHT.1)
3AD Φ-15
(SHT 1)
PS
SATSI
FTA19
SELFTA
ACCSB
SUB/ADD
KD1
ENSAT
CJ25
L19
L20
3,4 ACC ADDR Φ-5
(SHT.6)
3,4 ACC R/W
(SHT 6)
FORCE ZERO
SEL RAM
1,2 ACC ADDR Φ-5
(SHT 6)
1,2 ACC R/W
(SHT.6)
ACC 3-20
(SHT.4)
(SHT.6)
EN1
ENR
(SHT 6)

MODEL 660 (SN 300 -)
DIGITAL RANGE TRANSLATOR
F.I.R. (B) SECTION, PG. 2
100-SC-8314
SHEET 5 of 6 REV E
2-80

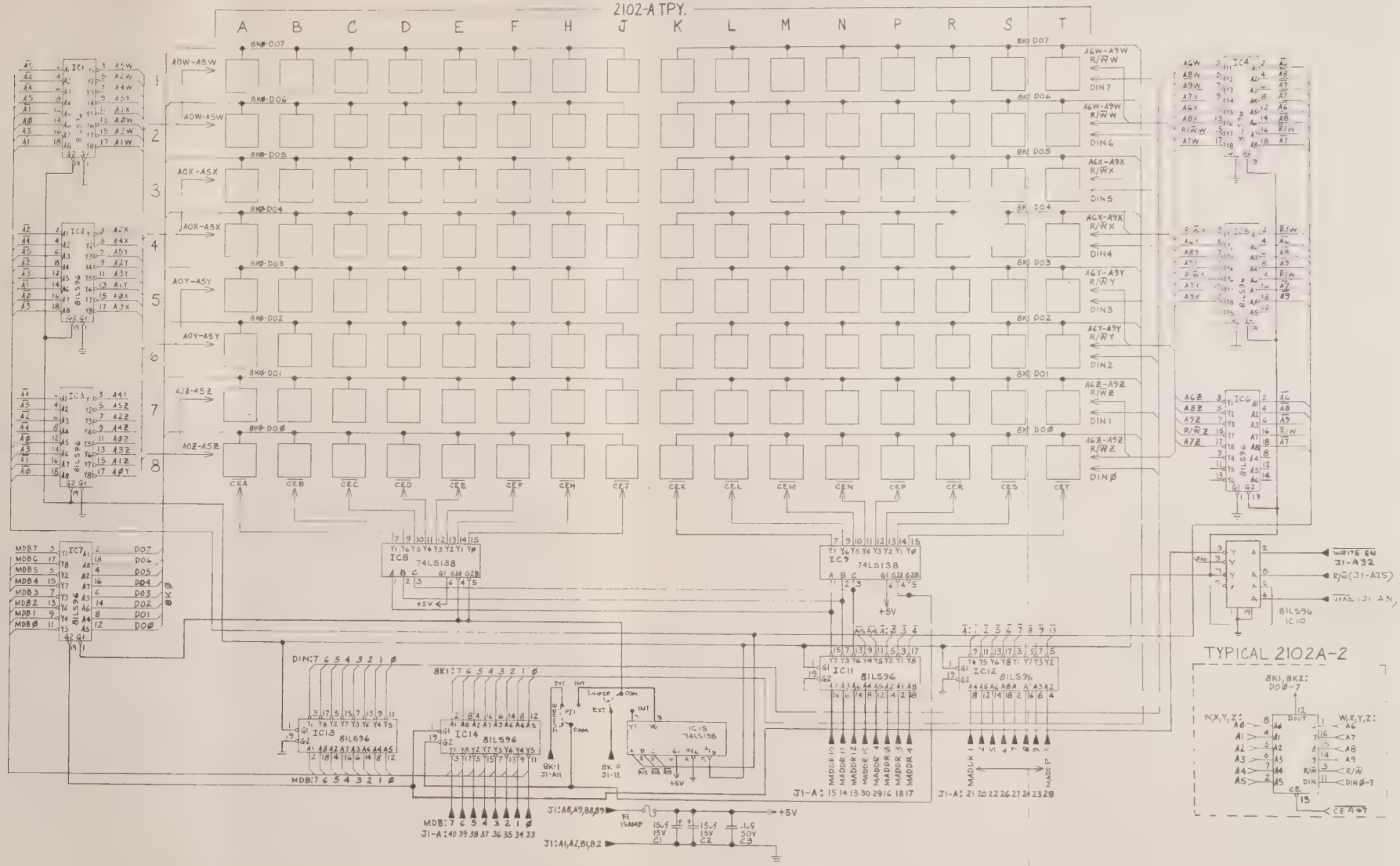
nicolet scientific



MODEL 660 (SN 300 -)
DIGITAL RANGE TRANSLATOR
FILTER (A & B) TIMING
100-SC-8314
SHEET 6 of 6 REV E
2-80

NO EXT BOARD AVAILABLE FOR MAIN MEMORY
MUST USE ROCKLAND SPECIAL TEST UNIT

nicolet scientific



OK DAC IC 6

+1.2	0	1	20 0	+6.2
+1.9	0	2	0	0
+2.5	0	3	0	+15
+2.3	0	4	0	0
+2.3	0	5	0	-2
+2.1	0	6	0	-2
+2	0	7	0	0
+1.9	0	8	0	0
1.7	0	9	0	+6.2
1.2	0	10	0	0
1.2	0	11	0	-15.3
.2	0	12	0	5

D/A CONVERTER

NICOLETT
660A

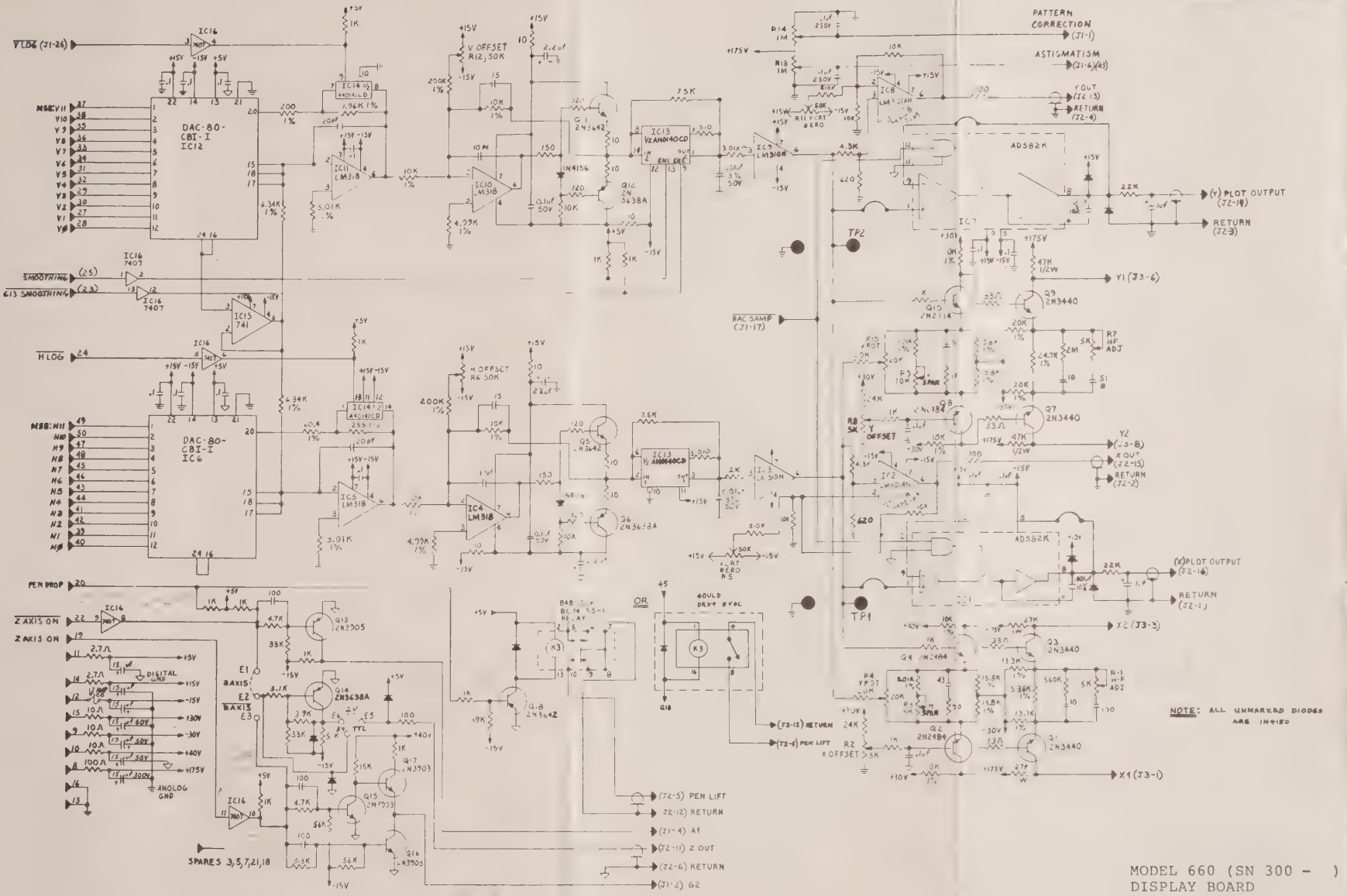
OK DAC IC 12

+1.4	0	1	20 0	+6.2
+1.6	0	2	0	0
+.7	0	3	0	+15
+1.2	0	4	0	0
+3.2	0	5	0	-1.7
+1.5	0	6	0	-1.6
+3	0	7	0	0
+.9	0	8	0	0
1	0	9	0	+6.2
1	0	10	0	0
+1.1	0	11	0	-15
.50	0	12	0	+5

D/A CONVERTER

NICOLETT
660A

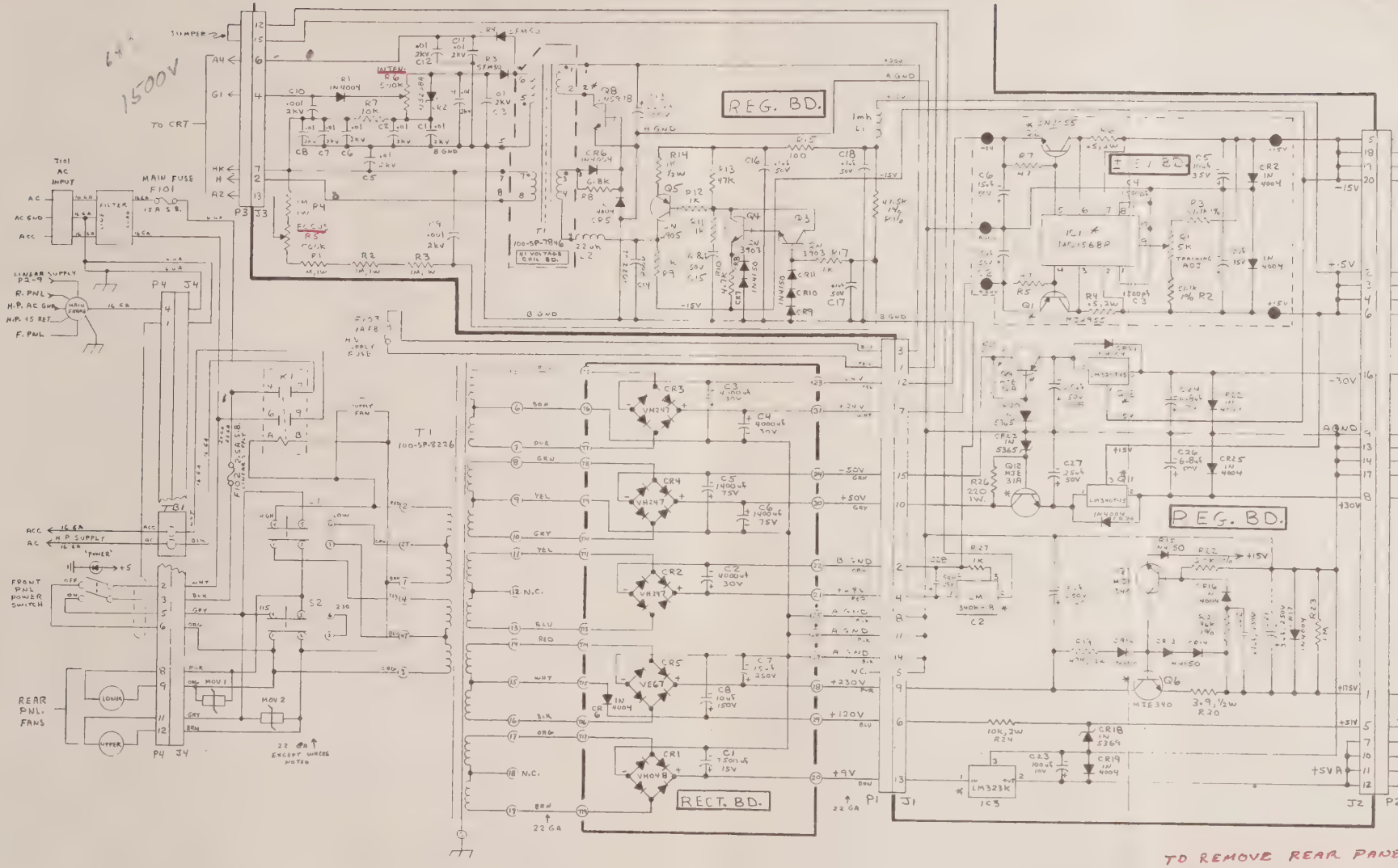
nicolet scientific



MODEL 660 (SN 300 -)
DISPLAY BOARD
100-SC-8322
SHEET 1 of 1

REV E
2-80

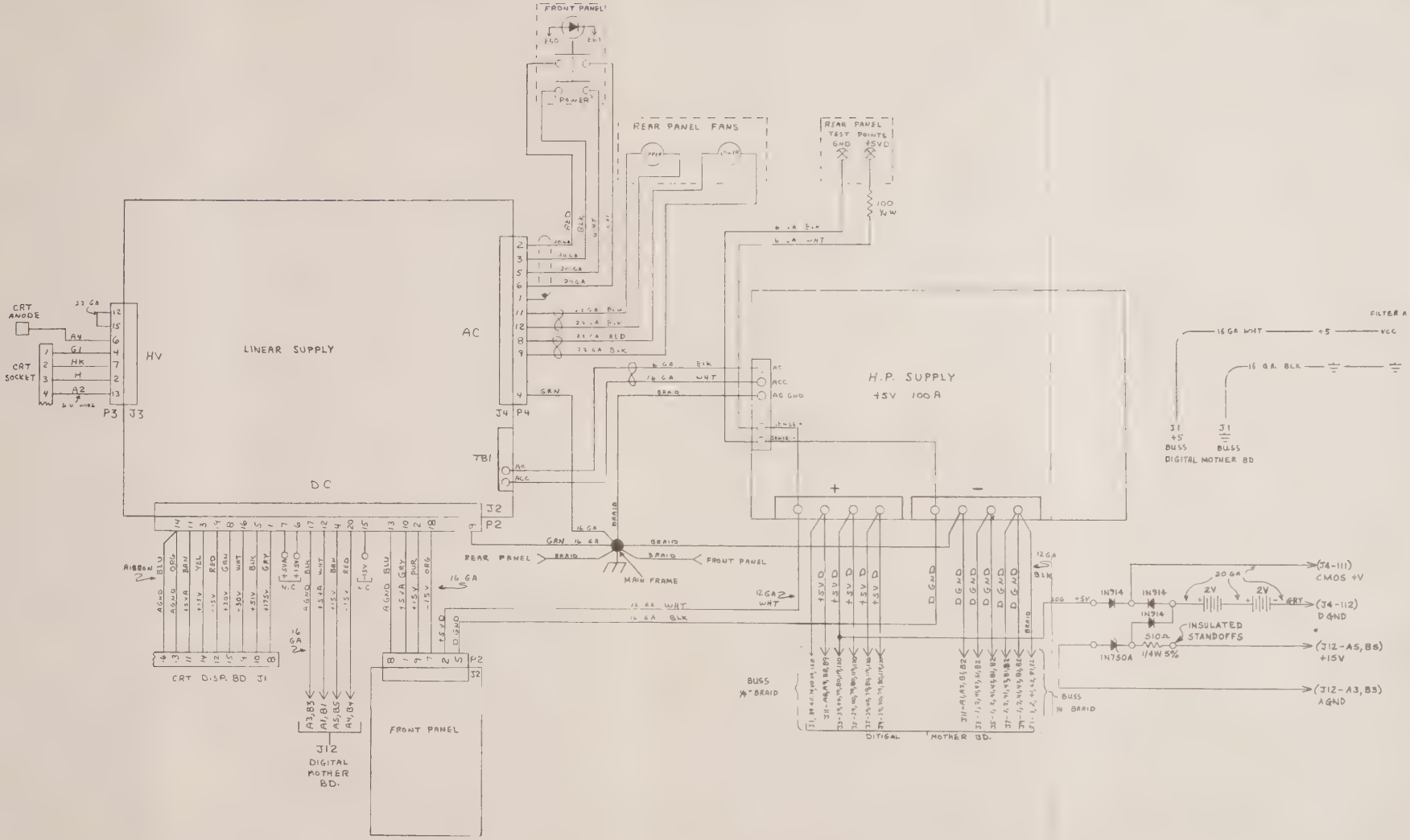
nicolet scientific



MODEL 660 (SN 300 -)
LINEAR SUPPLY
100-SC-8236
SHEET 1 of 1

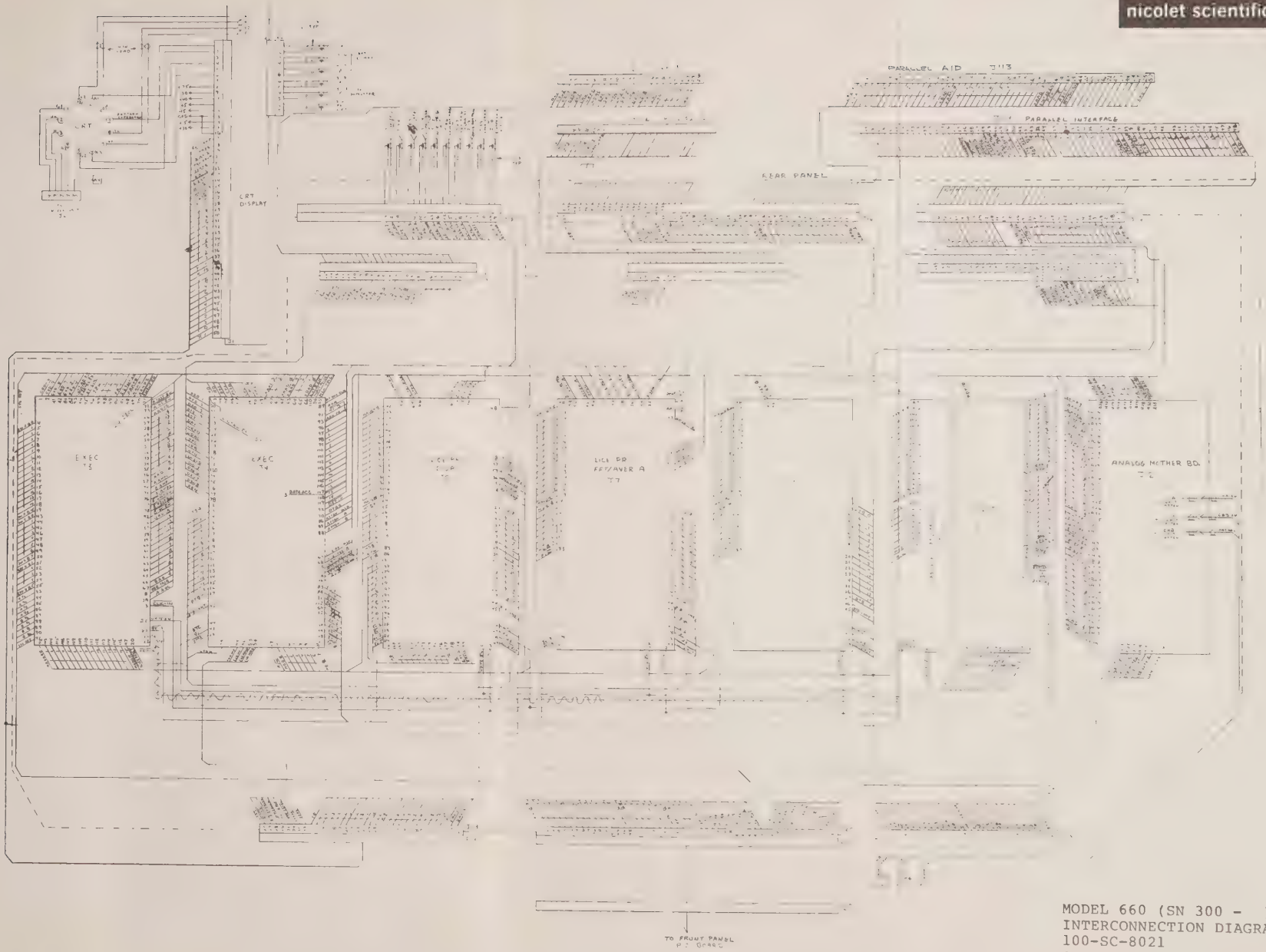
REV E
2-80

nicolet scientific



MODEL 660 (SN 300 - ·)
INTERCONNECTION DIAGRAM
100-SC-8021
SHEET 1 of 2 REV E
2-80

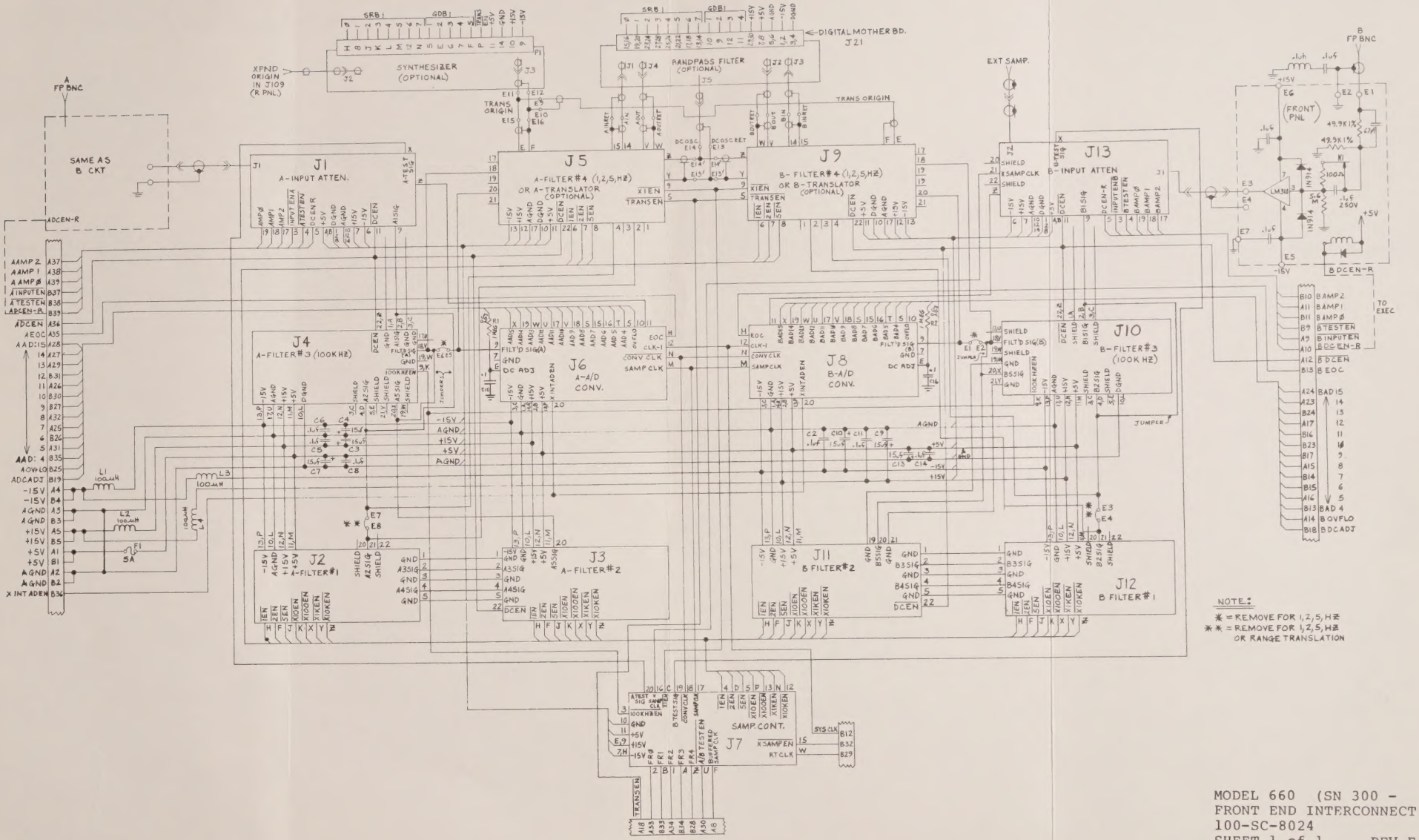
nicolet scientific



MODEL 660 (SN 300 -)
INTERCONNECTION DIAGRAM
100-SC-8021
SHEET 2 of 2

REV E
2-80

nicolet scientific



MODEL 660 (SN 300 -)
FRONT END INTERCONNECT
100-SC-8024
SHEET 1 of 1 REV E
2-80

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